Experimental techniques for turbulent Taylor–Couette flow and Rayleigh–Bénard convection
Invited Article

Experimental techniques for turbulent Taylor–Couette flow and Rayleigh–Bénard convection

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Abstract
Taylor–Couette (TC) flow and Rayleigh–Bénard (RB) convection are two systems in hydrodynamics, which have been widely used to investigate the primary instabilities, pattern formation, and transitions from laminar to turbulent flow. These two systems are known to have an elegant mathematical similarity. Both TC and RB flows are closed systems, i.e. the total energy dissipation rate exactly follows from the global energy balances. From an experimental point of view, the inherent simple geometry and symmetry in these two systems permits the construction of high precision experimental setups. These systems allow for quantitative measurements of many different variables, and provide a rich source of data to test theories and numerical simulations. We review the various experimental techniques in these two systems in fully developed turbulent states.

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(Some figures may appear in colour only in the online journal)

1. Introduction to the types of flow

1.1. Turbulent Taylor–Couette flow
Taylor–Couette (TC) flow systems consist of a working fluid confined between two coaxial, differentially rotating cylinders (inner cylinder—IC and outer cylinder—OC) with radii \( r_i \) and \( r_o \). The cylinders are able to rotate independently from each other with an angular rotation rate of \( \omega_i \) for the IC and \( \omega_o \) for the OC. The relative rotation of the cylinders can be in either the co-rotating or counter-rotating direction. The volume between cylinders is enclosed by two
Figure 1. A schematic of a TC apparatus. Two concentric cylinders can rotate independently. The rotation rates of the IC and OC are \( \omega_i \) and \( \omega_o \), and the radii are \( r_i \) and \( r_o \). The gap is defined as the volume where the working fluid is, which has gap-size \( r_o - r_i \), and height \( L \).

parallel lids with a distance ‘L’ between them. See figure 1 for a schematic illustration of a TC apparatus.

The TC is most conveniently described in a cylindrical coordinate system \((r, \phi, z)\) between the IC radius \( r_i \) and the OC radius \( r_o \). The aspect ratio is given by \( \Gamma = L/d \), where \( L \) is the cell height and the gap width is \( d = r_o - r_i \). Here we choose the control parameters in the TC system as the Taylor number \((Ta)\), the ratio of the angular velocities \((a)\) and the radius ratio \((\eta)\) which are defined as follows:

\[
Ta = \frac{1}{4} \sigma (r_o - r_i)^2 (r_i + r_o)^2 (\omega_i - \omega_o)^2 \nu^{-2},
\]

\[
a = -\omega_o/\omega_i,
\]

\[
\eta = r_i/r_o.
\]

In general, we define the angular velocity of the IC \( \omega_i \) as positive, whereas the angular velocity of the OC \( \omega_o \) is either positive (co-rotation, \( a < 0 \)) or negative (counter-rotation, \( a > 0 \)). The dimensionless control parameters \((Ta, a)\) of the system can also be expressed in terms of the IC and OC Reynolds numbers

\[
Re_i = r_i \omega_i d/\nu
\]

\[
Re_o = r_o \omega_o d/\nu,
\]

where \( \nu \) is the kinematic viscosity.

One of the response parameters is the degree of turbulence of the flow in the gap of the cylinders—i.e. quantified by the wind Reynolds number of the flow, which measures the amplitudes or fluctuations of the \( r \)- and \( z \)-components of the velocity field \((ur\), and \( uz\)). Since the time-averaged wind velocity is generally zero—i.e. \( \langle ur \rangle \simeq 0 \) and \( \langle uz \rangle \simeq 0 \) for only IC rotation, the standard deviation of the wind velocity is mostly used to quantify the wind Reynolds number:

\[
Re_w = \frac{\sigma_{uw} (r_o - r_i)}{\nu}.
\]
Figure 2. A sketch of Rayleigh–Bénard convection in a cylindrical cell with unit aspect ratio. The red-shaded areas of the cell show regions with hot fluid, while the blue areas indicate cold fluid. The arrows give the direction of the LSC.

$\sigma_{u_w}$ could be chosen to be the standard deviation of either the radial or axial velocity. Here we mostly use the radial velocity component.

Another response parameter is the torque $\tau$ to maintain the rotation of the IC at a constant angular velocity. The dimensionless form of the torque is defined as

$$G = \frac{\tau}{2\pi \ell \rho_{\text{fluid}} v_t^2}, $$

where $\rho_{\text{fluid}}$ is the density of the fluid and $\ell$ the height of the cylinder. Another frequently used parameter to represent the data is the friction coefficient on the IC

$$c_f = \frac{\left(1 - \eta \right)^2}{\pi} \frac{G}{Re_i^2} [1].$$

Based on the analogy between TC and Rayleigh–Bénard (RB), Eckhardt, Grossmann and Lohse (hereafter referred to as EGL) [2] defined a non-dimensionalized transport quantity as \( N_{\text{u,}} \) \('\text{Nusselt number}' in terms of the laminar transport,

$$N_{\text{u,}} = \frac{\tau}{2\pi L \rho_{\text{fluid}} J_{\text{lam}}^\omega},$$

where $J_{\text{lam}}^\omega = 2\nu r_i^2 r_o^2 (\omega_i - \omega_o)/(r_o^2 - r_i^2)$ is the conserved angular velocity flux in the laminar case. With this choice of control and response parameters, EGL derived a close analogy between turbulent TC and turbulent RB flow, building on the Grossmann and Lohse (GL) theory for RB flow [3–5].

With respect to flow instabilities, flow transitions, and pattern formation, TC flow is well-explored, and it displays a surprisingly large variety of flow states beyond the onset of instabilities [6–12]. Here we focus on TC flow in fully developed turbulent states. The parameters $Ta$ and $N_{\text{u,}}$ in the TC system are analogous to $Ra$ and $Nu$ in RB flow (to be discussed below). In the TC flow, one would like to characterize the flow structures, and measure the variables such as the angular velocity of the flow (transport quantity), the radial velocity (wind velocity), the global torque, and the local turbulent angular velocity flux.

1.2. Turbulent RB convection

The typical RB system consists of a working fluid confined between a cold top and a warm bottom plate [13–15]. As shown in figure 2, once the convective thermal turbulence has been well developed in the system, two types of boundary layers coexist near the top and bottom plates: one kinematic boundary layer and one thermal boundary layer. Thermal structures, like plumes, are generated and detached from the boundary layers, and then move into the bulk fluid under the effects of the buoyancy. During the rising and falling of thermal plumes,
the self-organization of the plume motion will result in a large-scale mean flow of the system, i.e. the large-scale circulation (LSC). The convective motion dramatically enhances the heat transport through the fluid layer, and understanding its nature is of fundamental interest and of great importance in the study of turbulent RB systems. The global heat transport is usually measured in terms of the Nusselt number $N_u$, defined as:

$$N_u = \frac{J}{\lambda f \Delta / H},$$

where $J$ is the heat-current density across a fluid layer of thermal conductivity $\lambda f$ with height $H$ and with an applied temperature difference $\Delta$.

The dynamics of the system depends on the turbulent intensity and the fluid properties. These are characterized, respectively, by the Rayleigh number $Ra$ and the Prandtl number $Pr$, defined as:

$$Ra = \frac{\alpha g H^3 \Delta}{\nu \kappa}, \quad \text{and} \quad Pr = \frac{\nu}{\kappa},$$

where $g$ is the acceleration due to gravity, and $\alpha$, $\nu$ and $\kappa$ are the thermal expansion coefficient, kinematic viscosity, and thermal diffusivity of the convecting fluid respectively, for which the Oberbeck–Boussinesq approximation is considered as valid, i.e. the fluid density $\rho_{\text{fluid}}$ is assumed to depend linearly on the temperature $T$, $\rho_{\text{fluid}} = \rho_0 [1 - \beta (T - T_0)]$, with $\rho_0$ and $T_0$ being reference values. In any laboratory experiments, a lateral sidewall is indispensable, so a third control parameter, the aspect ratio $\Gamma_1$, enters the problem. It is defined as the lateral confinement over the cell height. For a cylindrical cell (as shown in figure 2) $\Gamma_1 \equiv D / H$, where $D$ is the diameter of the cell.

In the RB flow, one would like to characterize the flow structures, and measure the variables such as the temperature and velocity of the fluid particles, and global and local heat flux.

2. Experiments in turbulent TC system

2.1. Flow visualization—turbulent flow structures

Flow visualization techniques are particularly useful in studying hydrodynamic systems which form spatial patterns due to secondary flows, for example Taylor vortices in TC flow [16]. There are many flow visualization techniques available, and here we only discuss the techniques which use anisotropic particles and small bubbles.

Matisse and Gorman [16] have described the flow visualization technique using anisotropic particles. The idea is to use particles dispersed in the working fluids to reflect the flow structures. Anisotropic particles are desirable to be used as they align themselves with the flow. Kalliroscope flakes are widely used for this purpose. Generally, Kalliroscope flakes are made from guanine, with a size of a few micrometres and a density of around 1.62 g cm$^{-3}$ [16]. In order to use this method, it is desirable to match the density of the flakes to the working fluid to reduce the sedimentation effect of the flakes. This visualization method has been widely used in the TC community to study the flow instability, pattern formation, chaos, and transition to turbulence (see e.g. [7, 8, 17, 18]).

Here, we show an example of the flow visualization using Kalliroscope flakes in the Twente turbulent Taylor–Couette (T$^3$C) facility. A detailed description of the T$^3$C system can be found in [20]. The IC has a radius of 20.0 cm and the OC radius is 27.9 cm. The 7.9 cm gap is vertically confined by the top and bottom plates with a distance of 92.7 cm. The experimental setup for the flow visualization is shown in figure 3(a). The fluid inside the TC gap is seeded with the reflective flakes and illuminated by a laser sheet. A high-speed camera is aligned...
Figure 3. Figure reproduced from [19]. (a) A photograph of the experimental setup for flow visualization in the T'C system. The flow is seeded with reflective Kalliroscope flakes and illuminated by a laser sheet. (b), (c) High-speed imaging snapshots show the evolution of the turbulent structures when the IC starts to rotate. On the left side is the IC wall, and on the right side is the OC wall. The distortion due to the curved surface is not corrected for these snapshots.

perpendicular to the light sheet to image the reflected light from the flakes. In this experiment, the OC is stationary, and the IC starts to rotate. As shown in figures 3(b) and (c), the recordings reveal the turbulent structures that emerge from the IC and their eventual penetration to fill the entire fluid layer.

It has been found that inertial particles in turbulence do not fully follow the fluid motion and distribute inhomogeneously within the turbulent flow, leading to clustering or preferential concentration [21,22]. Light particles tend to cluster in high vorticity regions of the turbulent flow [23]. One can take advantage of this property of the light particles to visualize the structures of turbulent Taylor vortices. Van Gils et al [19] visualized the flow structures in a turbulent TC flow with low-concentration small bubbles (the gas volume concentration is less than 0.1%). The left panel of figure 4 shows a snapshot of the high-speed recording of the distribution of the bubbles (the bubble diameter is less than 1 mm). As observed in the figure, these small bubbles clearly do not distribute uniformly in the flow. Most of the bubbles aggregate in the centre of the vortices, even at this very high Reynolds number ($Re_i = 1.2 \times 10^6$). Using image analysis techniques, van Gils et al [19] measured the dominant axial wavelength as explained in the figure caption.

Special attention must be paid to this technique. Many studies have suggested that the injection of bubbles can induce drag reduction in TC flows. One mechanism of the drag reduction is that the rising bubbles destroy the coherent structures and induce drag reduction [24]. In order to avoid the modification of flow structures in TC flow due to the small bubble injection, one has to carefully control the experimental conditions for flow visualizations: the rising velocity of the bubbles must be significantly smaller than the flow velocity fluctuations themselves. The volume fraction of the bubbles must be very small to have negligible effects on the flow field. In the experiments of van Gils et al [19], the typical rms velocity of the liquid flow is $\sim 6$ m s$^{-1}$, which is much larger than the rising velocity of the mm-sized bubbles (about 0.3 m s$^{-1}$), and the volume fraction of the bubbles is much smaller than 0.1%. The global torque (drag) measurement also confirmed that the change of the drag due to the bubble injection in this case is not detectable.
With increasing Reynolds number of the system, the TC system enters highly turbulent regimes. It becomes more difficult to visualize the flow structures with the Kalliroscope flakes or small bubbles. One reason is that the time scale of the velocity fluctuations is very short, and so the flakes/bubbles cannot respond to the rapid variation of the velocity fluctuations. Secondly, the flow structures might be unstable in time, and the transitions between different flow states would be too fast for visualization using the Kalliroscope flakes or small bubbles.

Although flow visualizations provide a global picture of the flow structures in TC flow, quantitative velocity measurements are crucial for studying turbulent TC flow. Many techniques are available to measure the flow velocity inside the TC gap. Below, we review a few intrusive and non-intrusive techniques.

2.2. Intrusive techniques for flow velocity measurements

2.2.1. Pitot tubes. A Pitot tube is a pressure-based measurement instrument, which is widely used to measure air/gas velocities in industrial applications and the airspeed of an aircraft. It is chiefly used to measure the local velocity at a given point in the flow stream as shown in figure 5. The measurement principle is based on Bernoulli’s equation: \( p_i = p_s + \frac{1}{2} \rho V^2 \), or \( V = \sqrt{2 \times (p_i - p_s)/\rho} \). The key requirements for the application of this method are: the Reynolds number is sufficiently large that the viscosity does not play a role; and the probe is small enough to satisfy the conditions of strict applicability of the Bernoulli equation. One has to carefully control the experimental conditions to obtain reliable results. For more details about the Pitot tube technique, we refer to [25].

In 1933, Wendt [29] performed velocity profile measurements in a TC apparatus using a Pitot tube at various \( Re \) numbers (\( Re_i \) and \( Re_o \)) up to \( 10^5 \), and found that technique was
Figure 5. A schematic illustrating the measurement principle of a Pitot tube. The diameter of the tube is typically of order mm.

Figure 6. Angular momentum normalized by angular momentum of the IC versus radius for a variety of Reynolds numbers. Measurements with a Pitot tube [26]: squares—$Re_i = 1.8 \times 10^4$, circles—$Re_i = 5.4 \times 10^4$, the radius ratio of the apparatus $\eta = 0.724$. Triangles from Smith and Townsend [27] measured with a hot-film probe at $Re_i = 5 \times 10^5$, and $\eta = 0.666$. Blue stars are the data from van Gils et al [28] measured with laser Doppler anemometry (LDA) at $Re_i = 1.0 \times 10^6$.

reliable and robust. Lewis [26] and Lewis and Swinney [30] carried out the angular velocity momentum profile measurements in the bulk region of the TC gap using the Pitot tube (United Sensor PCC-12-KL) technique at IC $Re_i$ up to $5.4 \times 10^5$ at the pure IC rotation case. The pressure difference was measured with a simple U-tube and ruler. An uncertainty of 3% in their velocity measurements at low fluid speeds resulted from the accuracy of measuring small height differentials in the U-tube. At higher rotation rates an uncertainty of 1% was determined by the limits of the technique [26]. Figure 6 shows the measured angular momentum profiles as a function of the normalized radial position, the open symbols are the measured data using the Pitot tube. They managed to measure the small amplitude of the angular velocity gradient in the bulk region of the turbulent TC flow. The measurements were only performed in the bulk region, due to the limitations in the probe size and measurement technique [26, 30].

2.2.2. Thermal anemometry. The thermal anemometry technique is widely used to measure rapidly varying velocity fluctuations with a good spatial and temporal resolution. Two types of sensors frequently used for flow velocity measurements are the hot-wire probe and hot-film probe. Hot-wire probes are generally used in gas flow measurements and hot-film probes are generally used in liquid flows. The sensors are typically made from materials like platinum or nickel, which have high specific resistivity, high temperature coefficient of resistance, small heat capacity and high tensile strength.

The measurement principle is based on the change in heat transfer of a small heated sensor exposed to the fluid motion. The change in the sensor resistance can be used to generate a measurable signal. In this technique, three operating modes are possible [25, 31]: constant-current anemometer (CCA), constant-temperature anemometer (CTA) and
Figure 7. Data from [32]. A typical calibration curve for a cylindrical hot-film probe: (a) polynomial curve fitting, and (b) power law curve fitting. The velocity values were measured using laser Doppler anemometry (LDA) to be discussed below.

Figure 8. A simplified sketch on a cylindrical hot-film probe inside a TC facility, left: vertical cross-section, right: top view.

constant-voltage anemometer (CVA). Since the constant-temperature mode is frequently used, sometimes hot-film and hot-wire probes are called CTA probes.

The advantages of the thermal anemometry technique include a good frequency response (400 kHz is feasible), a wide velocity range (from low subsonic to high supersonic flow), a high signal to noise ratio, continuous signals, and the possibility of simultaneous temperature measurements [31].

A CTA probe does not directly provide the flow velocity ($V$). The acquired signal is usually a voltage signal ($E$), and one has to calibrate the probe to convert the voltage signal into a velocity signal. The two functionals used for the calibration are the polynomial curve fitting $V = C_0 + C_1 E + C_2 E^2 + C_3 E^3 + C_4 E^4$ and power law curve fitting $E^2 = A + BV^n$ (King’s law), where $C_0, C_1, C_2, C_3, C_4, A, B$ and $n$ are the corresponding fitting parameters. Figure 7 presents an example of the calibration curves using polynomial and power law fits in a turbulent water flow at low velocities.

The thermal anemometry technique has been employed for velocity measurements in turbulent TC flows [26, 27, 30, 33]. The most commonly used probe type was the hot-film probe, and both cylindrical and conical probe geometries have been used for the measurements. Typically cylindrical probes have a dimension of ~1 mm in length and 100 µm in diameter, and conical ones have a diameter of ~1.5 mm. Cylindrical probes are more sensitive and conical ones are more rigid. Figure 8 shows a sketch of a cylindrical hot-film probe mounted in a TC apparatus.
Let us assume the flow to be in the following condition: the mean flow is in the azimuthal direction \( \langle u_\phi \rangle_t \), and the time-averaged radial and axial velocities are negligible \( \langle u_r \rangle_t = \langle u_z \rangle_t = 0 \). The hot-film anemometer can only measure the magnitude of the velocity, not the individual components. In the configuration shown in figure 8, the measured velocity amplitude is

\[
    u_{\text{eff}} = \sqrt{u_\phi^2 + u_r^2},
\]

where the \( u_\phi \) and \( u_r \) are azimuthal and radial velocity components respectively. In turbulent TC flow with only IC rotation, the flow satisfies the following conditions: \( \langle u_\phi \rangle_t \) is the dominant component, and the velocity fluctuations are much smaller than the mean flow velocity, i.e.

\[
    \langle u_\phi \rangle_t \gg \langle u_r \rangle_t, \quad (12)
\]

\[
    \langle u_\phi \rangle_t \gg \sqrt{\langle u'_\phi^2 \rangle_t}, \quad (13)
\]

where \( u'_\phi = u_\phi - \langle u_\phi \rangle_t \). In this case, the azimuthal velocity can be measured using a hot-film anemometer with a good accuracy \( u_{\text{eff}} \simeq u_\phi \). Figure 6 shows a comparison of angular velocity profiles in turbulent TC flow at only IC rotation measured with a hot-film probe [27] and a Pitot tube [26]. A good agreement between the two methods is found in the bulk regime, and the measured results are in reasonable agreement with the results measured using laser Doppler anemometry—LDA (to be discussed below) [28]. The hot-film probe also allows the measurement positions to be closer to the inner and outer walls compared to the Pitot tube.

The situation becomes complicated when the OC is also in rotation. The azimuthal velocity could be very small for a counter-rotation turbulent TC flow, i.e. \( \langle u_\phi \rangle_t \simeq 0 \). The measured velocity fluctuations can no longer be assumed to be mainly contributed by the azimuthal component. We have already noted that the hot-film anemometer does not provide information on the direction of the velocity. The velocity fluctuations around a zero mean flow cannot be well quantified with the hot-film anemometry. In bubbly flow system, a flying hot-film anemometry technique is used to overcome this issue. The sensor used is mounted on a sled, which is translated into the flow at a constant linear velocity [34]. However, it is not straightforward to implement this idea in the TC geometry. For TC with gases as working fluids, one can implement multiple hot-wire probes to measure the velocity components and the velocity gradient [35].

2.3. Non-intrusive techniques for flow velocity measurements

The Pitot tube and hot-film probe techniques can be used to measure the flow velocity inside the TC gap, but they suffer from drawbacks. These techniques measure the magnitude of the velocity, not the individual components, and lack directional information when a single probe is used. In addition, the flow is altered since they are intrusive measurement techniques. This is not a serious issue for non-recirculating setups, like an open-ended water/wind tunnel, but it can be a severe consequence in closed systems, such as a rotating drum [36], RB convection [14], and in a TC apparatus. It is well known that these probes can induce vortices, either in the form of a von-Kármán vortex street or as a turbulent wake depending on the Reynolds number and the detailed geometry. In a rotating drum experiment, Sun et al [36] found that these vortices can survive a full revolution of the system. Of course, there are also some non-intrusive probe-based techniques for flow measurements, like flush-mounted shear stress probes on a surface [26, 30].

LDA [37], particle tracking velocimetry (PTV) [38, 39] and particle imaging velocimetry (PIV) [40, 41] are optical techniques, which are discussed below in detail. These methods, by
their very nature, are non-intrusive and do not disturb the measured flow under consideration. In addition, these techniques can measure the individual velocity components and have directional sensitivity.

The addition of seeding particles is imperative for these techniques, and one should check whether these particles can accurately follow the flow. In order to ensure this, the particle Stokes number, defined as the ratio between the characteristic response time of the particle and the characteristic flow time scale [21], should be small enough. From a practical point of view, the measurement errors for velocity are below 1% when the particle Stokes number is less than 0.1 [25]. The key question in a TC flow is whether the centrifugal force on the seeding particles has a major effect on the particle movements. In [28], the seeding particles (PSP-50 Dantec Dynamics) have a mean radius of $r_{\text{seed}} = 2.5 \, \mu m$ and a density of $\rho_{\text{seed}} = 1.03 \, g \, cm^{-3}$. We estimate the maximum velocity difference $\Delta v = v_{\text{seed}} - v_{\text{fluid}}$ between a tracer particle $v_{\text{seed}}$ and its surrounding fluid velocity $v_{\text{fluid}}$. This velocity difference is assumed for the drag force $F_{\text{drag}} = 6\pi \mu r_{\text{seed}} \Delta v$ to outweigh the centrifugal force $F_{\text{cent}}(r) = 4\pi r_{\text{seed}}^3 (\rho_{\text{seed}} - \rho_{\text{fluid}}) v^2 / (3r)$. We put in $v = 5 \, m \, s^{-1}$ as a typical azimuthal velocity in the TC-gap at the mid-gap radial position $r = 0.24 \, m$, with $\rho_{\text{fluid}} = 1000 \, kg \, m^{-3}$ as the fluid density and $\mu = 9.8 \times 10^{-4} \, kg \, ms^{-1}$ as the dynamic viscosity (water at 21°C). This results in a velocity difference $\Delta v = 2r_{\text{seed}}^2 (\rho_{\text{seed}} - \rho_{\text{fluid}}) v^2 / (9 \mu r) \approx 5 \times 10^{-6} \, m \, s^{-1}$, which is several orders of magnitude smaller than the typical flow velocity fluctuation of order $10^{-1} \, m \, s^{-1}$ and therefore the centrifugal force acting on the seeding particles used in van Gils et al [28] is negligible. In individual experiments, one has to carefully check the properties of the tracer particles and the flow properties to ensure that the particle velocity faithfully reflects the true velocity of the flow.

2.3.1. Laser Doppler anemometry. LDA, as its name suggests, is based on the Doppler effect, which is familiar to all of us having heard the sirens of an emergency vehicle passing by. The same phenomenon occurs with light, though the change in colour is generally not observable by eye. The difference between the original frequency and the scattered frequency is called the Doppler shift $f_d$. This shift depends on the velocity of the particle, the wavelength of the light, and the position of the observer with respect to the source. A way of measuring the velocity would be to measure this Doppler shift directly, this is in practice however very difficult because the frequency shift is only a tiny fraction of the original frequency. The most commonly used version of LDA is a so-called dual beam heterodyne configuration, which solves this problem. In this configuration two beams are focused in one position: the measurement volume. A particle passing through the measurement volume will scatter light from both beams. The frequency shift is different for each beam, because the velocity vector makes a different angle for each beam and the observer. We call the Doppler frequency of beam 1 as $f_{d1}$, and for beam 2 as $f_{d2}$. Because both $f_{d1}$ and $f_{d2}$ are small as compared to $f$, the scattered signals have a similar frequency. The Doppler frequency can now be found by measuring the frequency difference between two beams, which can be done with high accuracy with a photo detector. The Doppler frequency is given by [42, 43]:

$$f_d = \frac{2 \sin(\theta/2)}{\lambda} |v_k|, \quad (14)$$

where $\theta$ is the angle between the two laser beams, and $v_k$ is the component of the velocity that is along $k_1 - k_2$, where $k_i$ are the propagation vectors of the laser beams.

In order to add directional information one shifts the frequency of one original beam, and this is accomplished using a Bragg cell. More details about the LDA in general, the Bragg cell, and the fringe-model, can be found in, e.g., [42–44].
In most situations, the LDA laser beams travel through flat surfaces, as shown in figure 9(a). In this case, by invoking Snell’s law equation (14) can be simplified as:

\[
\frac{f_d}{2|v_k|} = \frac{\sin(\theta_a/2)}{\lambda_w} = \frac{\sin(\theta_w/2)}{\lambda_a},
\]

where the subscripts with ‘w’ denote quantities in water, and ‘a’ in air. Equation (15) only holds when the interfaces are flat; it is only then that \( \theta_a/2 \) is the incidence angle and \( \theta_w/2 \) the angle of refraction. The changing of the wavelength absorbs the difference in refractive index, i.e. \( \sin(\theta_w/2) / \sin(\theta_a/2) = \lambda_w / \lambda_a \). So in the case of flat interfaces, \( \theta_a \) can be directly calculated from the beam separation and the focal length, and the light frequency/wavelength \( \lambda_a \). The velocity can be quantified from the Doppler shift (equation (15)). The refractive indices of the container and the working fluid are not used in the calculation of the velocity.

When the interface surface is curved (see figure 9(b)), equation (15) is no longer valid anymore to transform \( \theta_w \) to \( \theta_a \). Therefore, the measured velocity needs to be corrected by multiplication with a correction factor, i.e. \( u_{\phi,\text{real}} = C_\theta u_{\phi,\text{measured}} \), and \( C_\theta \) is

\[
C_\theta = \frac{n_w \sin(\theta_a/2)}{n_w \sin(\theta_w/2)}. \tag{16}
\]

To determine the correction factor, one can theoretically derive the trajectory of the laser light, see e.g. [46]. However, this analysis becomes very complicated if the system has more than one interface (e.g. multiple OCs). Huisman et al [45] built a ray-tracer to correct for two parameters: the angle \( \theta_w \) for the beam pair in the horizontal plane, and the position of the foci of the beam pair in the horizontal plane, respectively. Using the geometrical parameters of T3C apparatus, Huisman et al [45] calculated the correction factor \( C_\theta \) (see equation (16)) as a function of the radial position \( r \), as shown in figure 9(c). Note that the focal position also changes due to the curved surface, and see Huisman et al [45] for further details.

The results obtained from the ray-tracer were verified by measuring a well-known solid body rotation state in TC flow, when the IC and OCs are both rotating at the same angular velocity \( \omega \). The fluid has a velocity \( u_\phi = \omega r \), and \( u_z = u_r = 0 \) after a sufficient developing time. As an example, the results of the azimuthal velocity versus radial position for \( \omega / (2\pi) = 2 \) Hz measured by Huisman et al are shown in figure 10. The measured velocity profile without corrections is shown in red squares, which is higher than the actual solid-body
Figure 10. Figure reproduced with permission from [45]. © 2012 Elsevier Masson SAS. All rights reserved. In red squares the uncorrected azimuthal velocity as a function of radial position is shown, and in blue dots the corrected azimuthal velocity. The black solid line is the theoretical flow profile \( u_\theta = \omega r \), the deviation from this profile is plotted with green dots, the corresponding scale is on the right. The theoretical and corrected measured profiles are found to be in good accordance.

\[
\rho' = \frac{r - r_i}{r_o - r_i}
\]

Figure 11. Data from [28]. Radial profiles of the azimuthal angular velocity measured at the middle height \( z/L = 0.5 \), plotted against the dimensionless gap distance \( \tilde{r} = (r - r_i)/(r_o - r_i) \) for \( Re_i = 0 \) and \( Re_i = 1 \times 10^6 \). (a) The time-averaged angular velocity \( \langle \omega(\tilde{r}) \rangle \), normalized by the angular velocity of the inner wall \( \omega_i \). (b) Standard deviation of the angular velocity \( \sigma_\omega(\tilde{r}) \), normalized by the angular velocity of the inner wall. The boundary layers at the cylinder walls are not resolved. The open circles correspond to LDA measurements, and the solid dots are the measured data obtained using the PIV technique (to be discussed below).

profile (the black line). The data points (blue circles) are found to agree well with the actual flow profile within 0.75% after applying the beam angle correction. The remaining small deviation could be due to various reasons, e.g. spatially inhomogeneous refractive index in the working fluid, optical misalignment or imperfection, among others. It is clear that the ray tracer correction is crucial for reliable velocity measurements in TC flow.

Figure 11 shows the radial profiles of the azimuthal angular velocity and its standard deviation at a very high Reynolds number \( (Re_i = 1 \times 10^6) \) measured using LDA. The flow inside the gap is in a highly turbulent state, and the LDA has proven to be an excellent tool for the measurements of turbulent velocity and its fluctuations in TC flow [28].

2.3.2. Particle image velocimetry. It has been shown that the PIV technique is a powerful tool for direct measurements of the flow field in turbulent flows. The flow is seeded with tracer
particles, whose movements are used to indicate the fluid velocity. The mean displacement of the particles in a small interrogation window is quantified by the cross correlation of two consecutive images with a known time interval. The particles must satisfy the various criteria for the specific experiment conditions, as discussed previously.

The advantages of the PIV technique are: robustness, high accuracy, and the ability to do an entire field measurement. The robust nature of PIV stems from its conceptual simplicity. It directly measures two variables: displacement and time increment, which are the fundamental variables for the definition of velocity [47]. In contrast, a hot-film anemometer measures rate of heat transfer from the heated element in the fluid, and LDA measures the Doppler shift of scattered light from tracer particles. For the details on PIV, we refer to [47, 48].

PIV is a powerful tool for the study of flow structures and velocity profiles in turbulent TC flow. However, it is not straightforward to implement PIV in TC. Here we describe two examples where the PIV has been employed in turbulent TC flows.

Huisman et al [49] measured the azimuthal and axial velocity components in highly turbulent TC flow of $Re$ up to $10^6$. The experiments were performed in the T3C facility [20] with gap width of 7.9 cm. They utilized the viewing ports in the top plate of the apparatus to measure the flow from above. The experimental setup is shown in figure 12(a) with the side and top views. The flow was illuminated from the side using a pulsed laser (Litron LDY 300 Series, dual-cavity, pulsed Nd : YLF PIV Laser System), creating a horizontal laser sheet. The working fluid (water) was seeded with 20 $\mu$m polyamide seeding particles, and was recorded using a high-speed camera (FastCam 1024 PCI), operating at 1024 pixel $\times$ 1024 pixel resolution at frame rate of $f = 1$ kHz. To have a $\Delta t$ (the time interval between two consecutive images) far smaller than $1/f$ ($f$ is the frame rate), the PIV system was operated in the double-frame mode, i.e. two particle images are recorded on two frames.

Figure 12(b) shows two image pairs, and the velocity map corresponding to these two images computed from the Lavision PIV software (DAVIS 7.3) is shown in figure 12(c).
The PIV measurements provided direct access to both the angular velocity \( \omega(\phi, r, z, t) = \frac{u_\phi(\phi, r, z, t)}{r} \) and the radial velocity \( u_r(\phi, r, z, t) \) simultaneously. The solid dots in figure 11 show the profiles of the angular velocity and its standard derivation from the PIV measurements. Excellent agreement is found from the LDA and PIV measurements. In addition, PIV measurements can provide the statistics of the radial velocity component, which will be discussed below. When measuring from the top view, one has to pay careful attention to the out-of-plane velocity components, which in this context is in the axial (vertical) direction. The ratio of the vertical velocity and the azimuthal velocity varies significantly for different rotation ratios \( a = -\omega_o/\omega_i \). For the pure inner rotation case, the azimuthal velocity component is 1–2 orders of magnitude larger than the vertical one, therefore the out-of-plane motion is not important. But in the counter-rotation situations, the vertical velocity could be as large as the azimuthal one. Here, one has to pay special attention to the choice of \( \Delta t \) (the time interval between two consecutive images) by checking the quality of the cross-correlation map in the individual cases.

The top view PIV measurements cannot provide the flow field in the radial and axial \((r, z)\) plane simultaneously. One has to view from the side to measure the flow structures in the \((r, z)\) plane. In RB flow, one can mount a square ‘jacket’ outside of the RB system to reduce the distortion due to the curved surface [50]. In TC flow, since the OC is usually in rotation, it is not desirable to place the system in a box. However, the image distortion due to the curved surface can be corrected by calibrations [41].

Ravelet et al [41] measured three components of the velocity using stereoscopic PIV in a vertical plane of TC flow at a Reynolds number of \( Re = 1.4 \times 10^4 \). The experimental setup is indicated in figures 13(a) and (b). The measurement plane was vertical, i.e. normal to the mean flow: the in-plane components are the radial \((u_r)\) and axial \((u_z)\) velocities, while the out-of-plane component is the azimuthal component \((u_\phi)\). The measurement plane was illuminated using a double-pulsed Nd : YAG laser with a light-sheet thickness of 0.5 mm. The tracer particles they used were 20 \( \mu m \) fluorescent spheres. The measurement plane was imaged from both sides with an angle of 60\(^\circ\) (in air) using two double-frame CCD (charge coupled device) cameras on...
Scheimpflug mounts. The field of view in their experiments was $11 \times 25 \text{ mm}^2$ as indicated in figure 13(b). They took special care regarding the calibration procedure, because the evaluation of the plane-normal azimuthal component heavily depends on the calibration [41]. They used a thin polyester sheet with printed crosses as a calibration target, and attached this target on a rotating and translating stage. The calibration target was first put into the light sheet and traversed perpendicularly to it. Typically, they took five calibration images with intervals of 0.5 mm. The raw PIV images were processed using DAVIS 7.2 (Lavision Inc.) using a multigrid algorithm, with a final interrogation area of $32 \times 32$ pixels and 50% overlap. Then, the three velocity components were reconstructed based on the two camera views. Figure 13(c) shows an example of the time averaged flow at $Re = 1.4 \times 10^4$ with only IC rotation. The strong secondary mean flow in the form of counter-rotating vortices is clearly visible in their measurements.

Overall, the PIV technique is an excellent tool for the measurement of the velocity profile, flow velocity structures, and statistics of velocity fluctuations in turbulent TC flow. One can also measure the local transport fluxes using PIV, which we will discuss below.

2.4. Global torque and local angular velocity flux

2.4.1. Global torque measurements. The global torque on the cylinders in a TC setup can be measured by monitoring the power of the driving motors [51]. However, the technique suffers from several drawbacks, since it does not account for the drag from the bearings and seals, the efficiency losses, and the drag from the end sections [52]. A better approach for the global torque measurements is to use a strain gauge load-cell. A strain gauge is a long length of resistor arranged in a zigzag pattern on a membrane. When the strain gauge is stretched, its resistance increases. The strain gauges are mounted in a frame and often in four pieces to form a full ‘Wheatstone Bridge’, as shown in figure 14(a). In this configuration, a downward bend stretches the gauges on the top and compresses those on the bottom. The variations of the resistances of the strain gauges are transferred into a voltage signal (sketched in figure 14(b)) and amplified to yield the output signal.

Here we use the T3C apparatus as an example of the torque sensor mounting and calibration. A metal arm, consisting of two separate parts, is rigidly clamped onto the drive axle and runs to the inner wall of the IC section. The split in the arm is bridged by a parallelogram load cell (see figure 14(c)). Calibration of the load cells is done by repeated measurements, in which a known series of monotonically increasing or decreasing torques is applied to the IC surface, as shown in figure 15(a). The IC is not taken out of the frame and is calibrated in situ. The torque is applied by strapping a belt around the IC and hanging known masses on the loose end of the belt, after having been redirected by a low-friction pulley to follow the direction of gravity. Figure 15(b) shows the calibration data for one load cell (LSM300, Futek Inc.) in the T3C system. The three independent calibration data nicely collapse. The line in the figure is the best fit of the data using third-order polynomial fitting. The errors between the measured data and the fitting curve are shown in figure 15(c). The majority of the fitting errors are less than 0.2 Nm. However, one should pay attention to the hysteresis of the load cell. These three data sets were measured with monotonically increasing the load. The data were found to be different with decreasing the applied torque, suggesting that the hysteresis is not negligible for the torque calibrations and measurements. The maximum deviation due to hysteresis is found to be around 2 Nm, which is 10 times larger than the fitting errors for monotonically increasing torque. To avoid hysteresis, it is desirable to perform experiments with monotonically increasing or decreasing torque. Otherwise, one should calibrate the load cell for both increasing and decreasing torque respectively.
Figure 14. (a) A sketch of a load cell and the strain gauges. (b) A sketch of the electrical circuit of the load cell. (c) Reproduced with permission from [20]. Copyright 2011 AIP Publishing LLC. Horizontal cut-away showing the load-cell construction inside the ICmid drum. The load cell spans the gap in the arm, connecting the IC drive axle to the IC wall.

Figure 15. Figure reproduced from [53]. (a) A photograph of the T3C showing that the load cells are calibrated in situ. (b) The calibration data for one load cell (LSM300, Futek Inc.) mounted in the T3C facility. The data were measured by monotonically increasing the applied load. (c) The corresponding fitting errors of the calibration.

Figure 16 shows one example of $Nu_{\omega}$ versus $Ta$ for various $a$ measured in the T3C facility by van Gils et al [54]. A universal effective scaling $Nu_{\omega} \propto Ta^\gamma$ with $\gamma \approx 0.38$ is clearly revealed. And this efficient exponent does not depend on the control parameter ‘$a$’. This corresponds to a scaling of $G \propto Re_i^{1.78}$ for the dimensionless torque [52], to $c_f \propto Re_i^{-0.24}$.
for the drag coefficient, and to $G \propto T a^{0.88}$ [54]. We refer readers to [28, 54–56] for further details about the global torque versus the control parameters.

2.4.2. Local angular velocity flux measurements based on velocity correlation. One also can indirectly obtain the torque value by measuring the correlation of the radial and angular velocity fluctuations. The condition is that the angular velocity (or azimuthal velocity) and radial velocity must be measured simultaneously at the same physical point, which can be realized using LDA [57, 58] or PIV [59].

In turbulent TC flow, the (total) angular velocity flux (convective + diffusion) is

$$J_\omega(\phi, r, z, t) = r^3(\omega_t - \nu \partial_r \omega_r).$$  \hbox{ \hfill (17)\hfill}$$

In the turbulent regime the convective term is the major contributor to the flux in the bulk [28]. The measurements of the angular velocity flux $J_\omega(\phi, r, z, t)$ in turbulent TC flow at the bulk regime can be simplified as

$$J_\omega(\phi, r, z, t) \sim r^3 \omega_t,$$  \hbox{ \hfill (18)\hfill}$$

which is made dimensionless by its value for the laminar non-vortical flow, $J_\omega^{\text{laminar}} = 2\nu r_t^2 \omega_t^2 (\omega_t - \omega_o) / (r_o^2 - r_i^2)$, giving the local ‘Nusselt number’ (see [2])

$$Nu_\omega(\phi, r, z, t) = J_\omega(\phi, r, z, t) / J_\omega^{\text{laminar}}.$$  \hbox{ \hfill (19)\hfill}$$

This $Nu_\omega(\phi, r, z, t)$ is the local convective angular velocity flux. One must note that an additional axial average is required to have the exact relation between $Nu_\omega$ and the global torque $\tau$ that is required to obtain the IC at constant velocity ([2]),

$$\tau = 2\pi L p J_\omega^{\text{laminar}} (Nu_\omega)_{\phi, z, t}.$$  \hbox{ \hfill (20)\hfill}$$

Huisman et al [59] directly measured the local angular velocity flux in the turbulent TC flow using PIV. The measurements were done in the case of IC rotation at mid-height of their TC apparatus $z = H/2$. For three different rotation rates $\omega_o / (2\pi) = 5$ Hz, 10 Hz and 20 Hz (corresponding to $Ta^3 = 3.8 \times 10^{11}, 1.5 \times 10^{12}$ and $6.2 \times 10^{12}$) they measured the local velocity from the top of their TC apparatus. One example of the velocity map is shown in figure 12(c). The local velocity $\vec{u}$ is then decomposed into the angular velocity $\omega(\phi, r, z, t)$ and the radial velocity $u_r(\phi, r, z, t)$ by aligning the $\phi$ direction to $\langle \vec{u} \rangle$. The corresponding local $Nu_\omega(\phi, r)$ is obtained based on equations (18) and (19). In figure 17

Figure 16. Figure reprinted with permission from [54]. Copyright 2011 by the American Physical Society. $Nu_\omega$ versus $Ta$ for various $a$: a universal effective scaling $Nu_\omega \propto Ta^{0.38}$ is revealed.
we show several snapshots of $Nu_\omega(\phi, r)$ at mid-height $z = L/2$ for $Ta = 1.5 \times 10^{12}$. The quantity of $Nu_\omega(\phi, r)$ shows huge fluctuations, ranging from $+10^5$ to $-10^5$ and even beyond, whereas the time-averaged $\langle Nu_\omega(\phi, r, t) \rangle_{\phi,r,t} = 325$ is found to be very close to the globally measured value $Nu_\omega^{\text{glob}} = 326 \pm 6$ obtained from the load cell [54]. The instantaneous local flux fluctuations can thus be more than $\pm 300$ times as large as the time-averaged flux. Such large fluctuations have also been reported for the local heat-flux in RB flow [60], but in that case the largest fluctuations were only 25 times larger than the mean flux. The fluctuation amplitudes will of course depend on the control parameters of the system.

2.5. Two-phase (bubbly) TC flow

The injection of bubbles offers a new direction in the study of TC flows. There is huge interest in drag reduction, since it can lead to significant reduction of the fuel consumed by ships without adding contaminating substances into water [61]. The TC system is an ideal test bed for studying the drag that a surface experiences when moving in a bubbly flow [62–65]. The experimental challenges in bubbly TC flows are to locally measure and quantify the bubble size and distribution.

Although one can measure the bubble size and distribution with the particle tracking velocimetry technique in the very dilute (<1%) bubbly flow [32,66], the optical fibre technique is mainly used for bubble detection in bubbly flows [67–74]. The optical probe is a local phase detection device, which can distinguish between water and air. The light emitted by a light source is sent through one extreme of the optical fibre. When the light reaches the other extreme of the fibre (treated to have a specific geometry—the so called tip) a part of the light is reflected. The reflection coefficient depends on the refractive index of the phase in which the fibre tip is submerged. By measuring the amount of reflected light over time, one can obtain bubble statistics. For a detailed description of the working principle, we refer to [75].

In their experiments on bubbly TC flow, van Gils et al [76] used a custom made glass fibre probe (0.37 NA Hard Polymer Clad Multimode Fibre, Thorlabs Inc., diameter of 200 µm), whose tip has a smooth U-shaped dome, as shown in figure 18(a). A photodiode was used to measure the internally reflected light from the optical fibre. In the work of van Gils et al [76], the configuration of the optical fibre in the T³C system is shown in figure 18(b). The optical probe passed through a small hole of the OC with the fibre tip, and was placed inside the TC gap, and parallel to the azimuthal axis.

Figure 19(a) shows a snapshot of the bubble distribution in the T³C at $Re_l = 5.1 \times 10^5$ with stationary OC. Figure 19(b) shows the typical voltage signal from an optical fibre in a
bubbly TC flow. One can clearly see the voltage peaks with different heights and widths. In order to detect the bubbles, one can analyse the peaks with a magnitude above a threshold value. In the present example, the threshold value is selected to be 1 V. The voltage data points above the threshold value are assumed to correspond to the fibre tip immersed in air. The bubble mask built on this threshold value in the present case is shown with the solid blue lines. The width of the bubble mask is then measured by the residence time $T_{\text{bubble}}$ per specific bubble, and the bubble size and local gas concentration can be calculated, see [76] for further details. As shown in figure 19(b), a positive signal slope of the voltage signal reflects the dewetting process of the fibre tip, and the rewetting process is indicated by the voltage drop (negative slope). van Gils et al [76] also examined statistics of the rising and fall time, which measure the bubble-probe interaction time. We refer to [76] for further details.

3. Experiments in turbulent RB flow

3.1. Visualization

In the process through which people understand the world, qualitative knowledge is always before precise measurements and abstract theories. Throughout the history of fluid dynamics research, flow visualization has always been an important tool. It has also been used extensively in the fields of engineering, physics, medical science, meteorology, and oceanography.
Figure 20. Figure reproduced with permission from [83]. A schematic diagram of the experimental setup for the shadowgraph visualization. When a cylinder is used as the convection cell as shown in the figure, a square-shaped jacket made of flat glass plates is fitted round the sidewall of the cell. The jacket is filled with the same convecting fluid as in the cell. This greatly reduces the distortion effect to the images caused by the curvature of the cylindrical sidewall.

For turbulent RB systems, there are several techniques that have long been used to visualize the global and local flow structures, such as the chemical dying technique [77, 78], the planar-laser-induced-fluorescence (PLIF) technique [79, 80], the shadowgraph technique [81–84], and the thermochromic-liquid-crystal (TLC) technique [85–88]. In this section, we will discuss two visualization techniques: shadowgraph and TLC.

3.1.1. Shadowgraph. The key idea here is that the refractive index of the working fluid depends on temperature and thus parallel light rays either diverge or converge when passing through hot or cold fluids, respectively, and then form dark or bright areas. Figure 20 shows a typical experimental setup used for the shadowgraph visualization. The light source is a white light from a halogen light source. The light first passes through a pinhole in order to produce a point light source. A Fresnel lens then provides a uniform and collimated beam of light to shine through the cell. On the other side of the cell, the image of the flow is seen on a screen made of oil-paper and is captured by a CCD camera placed behind the screen (not shown in the figure). Because the light is not focused onto a specific plane, the projected image on the screen is simply an integration of refractive index contrast along the lightpath. To reduce the effect of light intensity inhomogeneity and other optical imperfections, the shadowgraph is usually divided pixel by pixel by a background image that is taken before the temperature gradient is imposed. The images obtained from the experiments are then rescaled and smoothed.

In the field of turbulent RB convection, shadowgraph is usually adopted to visualize the motion of thermal plumes and the LSC. For example, based on this technique, Xi et al [83] revealed that the LSC is a result of the self-organization of the plume motion [83] and Ahlers and co-workers discovered the torsional model of the LSC [91, 92]. Figure 21 shows two examples of shadowgraph visualizations. The left panel is taken from [89]. Thermal plumes are clearly visible from the picture: with hot plumes rising up at the left-hand corner of the bottom plate and cold plumes falling down from the right-hand corner of the top plate. The motion of the plumes at the same time gave us an indication of the existence of the LSC, which moved clockwise along the sidewalls of the cell and dragged the plumes along with it. The right panel of figure 21 is a shadowgraph image showing the spatial structure of the LSC [90]. When viewed from the positive x direction the flow is overall a single-roll structure, while the
Figure 21. Left panel reprinted with permission from [89]. Copyright 2003 by the American Physical Society. A shadowgraph visualization of the spatial distribution of thermal plumes at \( Ra = 6.8 \times 10^8 \) and \( Pr = 596 \) in a \( \Gamma = 1 \) cylindrical cell. The dark line hanging from the top plate is the thermistor with a stainless steel tube, which was used to measure the temperature at the centre of the cell. Right panel: figure taken from [90]. Shadowgraph images taken simultaneously by two CCD cameras at an angle of 90° in a horizontal plane at \( Ra = 8.3 \times 10^8 \) and \( Pr = 1032 \) in a \( \Gamma = 1 \) cell. The convecting fluid is dipropylene glycol in both cases.

Fluid goes up vertically within a narrow band near the central region of the cell when viewed from the positive y direction. The combination of the two views then suggests that the flow in the \( \Gamma = 1 \) cylindrical cell is a quasi-two-dimensional single-roll structure with a finite width of about half cell diameter.

A similar experimental setup for shadowgraph experiments is shown in figure 22(a). Light from a ‘point source’ is directed into the convection cell by a pellicle beam splitter. The light is then reflected from the cell bottom plate and returns to the optics tower, having passed twice through the fluid layer. It is refocused by the collimating lens and imaged by a lens onto a
3.1.2. The TLC technique. Here, the key idea of the TLC technique is that the peak wavelength of the scattered light of TLC microspheres changes with fluid temperature. Usually, the TLC microspheres are neutrally buoyant and hence can be used as passive tracers. Figure 23 shows a schematic diagram of the optical setup for the TLC visualization. A halogen photo optic lamp (S) is used as the source of white light. A concave mirror L1 and several condensing lenses (L2 and L3) are used to collect the light from S and focus it onto the central section of the cell. A sheet of light, generated by a diverging cylindrical lens L4 and then projected onto an adjustable slit, is passed through the cell. In the figure, a horizontal light sheet is generated [88] to visualize the horizontal flow near the top plate. By adjusting the position and orientation of the slit, one can also generate other kinds of light sheets, like a vertical sheet [87, 98]. A CCD camera is used to take streak photographs of the TLC microspheres. With short camera exposure time the photographs give the instantaneous temperature field, and with long exposure time they will in addition show the trajectories of the particles.

Using the TLC technique, Zhou et al [88] revealed how sheetlike plumes evolve morphologically to mushroomlike ones near the plates. As shown in the left panel of figure 24, the motion of TLC microspheres appears to emanate from certain regions or ‘sources’ with bluish colour, suggesting that hot fluid (plumes) moves upwards, impinging on and spreading horizontally along the top plate. Along the particle traces, the colour turns from blue to green and red, implying that the wave fronts are cooled down gradually by the top plate (and the top thermal boundary layer) as they spread. As they travel along the plate surface, sheetlike plumes collide with each other or with the sidewall. As different plumes carry momentum in different directions, they merge, convolute and form swirls (hence generating vorticity). As these swirls are cooler than the bulk fluid, they spiral away from the plate, and then merge and cluster together.

Based on the TLC technique, Zhou and Xia [97] further developed a tomographic reconstruction technique to construct the 3D image of thermal plumes from sequences of 2D images acquired near the top plate of the cell. They placed the convection cell on a translational stage. The cell traversed continuously, during which a series of photographs of TLC microspheres were recorded by a colour camera. In post-experiment analysis, 2D...
Figure 24. Left: reproduced with permission from [88]. Copyright 2007 by the American Physical Society. A image of TLC microspheres taken at 2 mm from the top plate in a water-filled $\Gamma = 1$ cylindrical cell with height 18.5 cm. The used particles have a mean diameter of 50 $\mu$m and density of 1.03–1.05 g cm$^{-3}$, and were suspended in the convecting fluid in very low concentrations (0.01 wt%). The peak wavelength of their scattered light changes within a temperature window divided as follows: $29^\circ$–$29.5^\circ$ for red, $29.5^\circ$–$29.7^\circ$ for green, and $29.7^\circ$–$33^\circ$ for blue. The camera exposure time is 0.77 s. Right: figure taken from [97]. Three-dimensional thermal plumes obtained near the top plate using the TLC tomographic reconstruction technique. Both measurements were made at $Ra = 2.0 \times 10^9$ and $Pr = 5.4$ in a $\Gamma = 1$ cell.

Figure 25. Figure reproduced from [99]. Left panel: a typical relationship curve for thermistor: temperature versus resistance. Right panel: the Wheatstone bridge. $R_v$ is used to balance the bridge.

horizontal cuts of cold plumes were extracted from the images taken in each run and a MATLAB script was used to reconstruct 3D thermal plumes from these extracted 2D cuts. The right panel of figure 24 shows an example of the reconstruction of 3D thermal plumes near the top plate. One noteworthy feature is that most plumes have a 1D structure, rather than a sheetlike shape: both the height and the width of this plume extend only to a few millimetres, while its length extends to a few centimetres. Based on this, Zhou and Xia [97] suggest that the so-called sheetlike plumes are really 1D structures and may be called rodlike plumes.

3.2. Temperature measurements

Temperature measurement is surely essential for thermal turbulence. Usually, a temperature sensitive resistor, like a thermistor, is used as a thermal probe. Most thermistors have a negative temperature coefficient, i.e. their resistance decreases with increasing temperature. A slight temperature change causes a rapid and pronounced change in electrical resistance. As shown in the left panel of figure 25, the relationship between absolute temperature $T$ (in Kelvins)
and thermistor resistance $R$ is well described by the equation proposed by Steinhart and Hart [100]:

$$\frac{1}{T} = a_0 + a_1 \ln R + a_2 (\ln R)^3,$$

(21)

where $a_0$, $a_1$, and $a_2$ are the fitting parameters. (The most general form of the applied equation contains a $(\ln R)^2$ term, but this is frequently neglected because it is typically much smaller than the other terms.) Thermistors are usually used to monitor the temperatures of the top and the bottom plates, as well as local temperatures in the cell, such as those within the thermal boundary layers [101, 102]. Fast and accurate measurement of the thermistor resistance gives us a direct reading of the precise local temperature fluctuations. To achieve this, a Wheatstone bridge is usually adopted. The right panel of figure 25 shows the circuit of an ac Wheatstone bridge. An ac source is often chosen rather than dc source for better sensitivity, more accuracy and less noise. Two arms of the bridge are precision resistors of resistance $R_0$. A variable resistor $R_v$ and the thermistor $R_{th}$ are connected to the other two arms. The ac source ($U_s$) supplied to the bridge is a sinusoidal wave with frequency much higher than sampling frequency. For a given value of the variable resistance $R_v$, the thermistor resistance $R_{th}$ can be calculated from the following equation:

$$R_{th} = \frac{R_0}{\frac{R_0}{R_0+R_v} \frac{U_{AB}}{U_s} = R_0},$$

(22)

where $U_{AB}$ is the voltage difference between points A and B. A lock-in amplifier is used to measure this voltage difference. Figure 26 shows an example of the temperature time series measured with a thermistor in the working fluid of water at the cell centre and near the sidewall at $Ra = 1 \times 10^{10}$ and $Pr = 4.3$. One can clearly see that the measurements can well resolve small amplitude fluctuations.

### 3.3. Velocity measurements

The conventional velocimetry techniques, such as hot-wire or hot-film anemometers, have been widely used to measure the velocity in various turbulent flows as discussed above. However, these techniques are usually operated with a constant-temperature circuit and hence they are
not ideal for use in thermal turbulence, like turbulent RB convection. On the other hand, optical techniques have developed greatly in the past several decades in the application of fluid flows because they can provide an insight into the flow structure. Combined with quantitative measurements of specific flow parameters such as velocity, density, pressure and temperature, they provide an accurate and complete picture of the flow. In this section, we discuss the applications of two popular optical velocimetry techniques in turbulent RB convection, i.e. LDA and PIV.

3.3.1. Laser Doppler anemometry. The principle of LDA has already been discussed above, and it is widely used to measure the velocity fluctuations in turbulent flows. In the past decade, LDA has been widely used in turbulent RB systems, see, e.g. experiments by Ashkenazi and Steinberg [103], Daya and Ecke [104], Qiu and Tong [105, 106], Shang and Xia [107] and Du Puits et al. [108, 109] among others. Figure 27 shows an example of the time series of velocity fluctuations at the cell centre measured with LDA [110]. The fluctuations of the velocity are nicely quantified with this technique.

3.3.2. Particle image velocimetry. The technique of PIV provides us with a convenient tool to directly visualize and measure the flow field of a particular plane of interest in three-dimensional turbulent systems [112]. The details of the PIV technique have already been discussed above. With the PIV technique one captures two consecutive two-dimensional (2D) images of the seed particles, using a CCD camera situated normal to an illuminating light sheet as shown in figure 28. Spatial correlations between the two images are then calculated to obtain information about the displacement of the particles, from which one obtains the velocity map. The main advantage of the PIV method is its ability to follow the motion of a 2D flow field. With the 2D time series data, one can obtain both the time-averaged and the dynamic.
Figure 28. Sketch of the setup for PIV measurement for a cylindrical RB cell. The PIV system consists of a dual neodymium-doped yttrium aluminum garnet (Nd:YAG) laser, lightsheet optics, a cooled CCD camera, and a synchronizer.

Figure 29. Figure reproduced with permission from [111]. Copyright 2005 by the American Physical Society. Instantaneous velocity vector maps taken in the plane of the LSC at \( Ra = 7.0 \times 10^9 \). The magnitude of the velocity is coded in colour and the length of the arrows in units of cm s\(^{-1}\). Instantaneous velocity map taken at (a) early time and (b) 20 s (about half of the oscillation period at this \( Ra \)) later.

properties of the 2D flow field. By measuring the velocity maps in different planes, one can examine the 3D structures and dynamics of the velocity field. The light sheet is generated using cylindrical lenses as indicated in the inset of the figure 28. A square-shaped jacket made of flat glass plate is fitted to the outside of the sidewall. The jacket is also filled with water, which greatly reduces the distortion effect to the PIV images caused by the curvature of the cylindrical sidewall [111]. Instantaneous image capture and high spatial resolution of the PIV allow the detection of spatial structures even in unsteady flow fields, see Sun et al [111, 113]. Figure 29 shows two examples of the instantaneous velocity maps, which reveal the spatial coherence of the thermal plumes and bulk velocity oscillation (see [111] for further details).
Recently, a high-resolution PIV technique has been employed to study the viscous boundary layer structures in turbulent RB convection [114, 115], as shown in figures 30(a)–(c). A very small area (shaded region in (a)) is illuminated with a thin light sheet (~0.2 mm) generated with the optics shown in figure 30(b). The time-averaged and instantaneous velocity maps at two different Ra numbers are shown in figures 30(c) and (d), respectively. Sun et al [114] found that, despite the intermittent emission of plumes, the Prandtl–Blasius-type laminar boundary layer description is indeed a good approximation, in a time-averaged sense, both in terms of its scaling and its various dynamical properties. Zhou and Xia [115] further showed that the velocity profiles agree well with the classical Prandtl–Blasius laminar boundary-layer profile, if they are resampled in the dynamical reference frames that fluctuate with the instantaneous velocity boundary-layer thickness as shown in figures 30(d) and (e).

3.4. Global heat transport and local heat flux

The global heat transport can be measured by monitoring the input heat-current from bottom plate. The main challenge is to reduce the heat leakage from the sidewall and bottom plate. Brown et al [116] provided detailed descriptions on the construction of the system to minimize the heat leakage. We refer the reader to [116] for more details on the global heat transport.
measurements. Many experiments on the global measurements of the Nusselt number were conducted with wide parameter range and great precision, see recent review articles by Ahlers et al [14], by Chilla and Schumacher [117] and by Xia [118] for further details.

Shang et al [60, 119, 120] directly measured the local convective heat flux, \( J(r, t) = \langle v(r, t) \delta T(r, t) \rangle /H/\kappa \Delta \), at different Ra and spatial positions \( (r) \). Here, \( \langle \ldots \rangle_t \) indicates an average over time \( t \). To obtain the local heat flux, one has to measure the local temperature fluctuation \( \delta T = T(r, t) - T_0 \) and local flow velocity \( v(r, t) \) at the same position simultaneously (here \( T_0 \) is the mean bulk temperature). As shown in figure 31, a small movable thermistor of 0.2 mm in diameter, 15 ms in time constant, was used to measure \( T(r, t) \). The local velocity was measured using a LDA. Simultaneous velocity and temperature measurements were realized using a multichannel LDA interface module to synchronize the data acquisition. A triggering pulse from the LDA signal processor initiated the acquisition of an analogue temperature signal. Figure 31(b) shows the measured time series of the temperature fluctuation \( \delta T(t) \) (top curve), vertical velocity (middle curve) \( v_z(t) \), and the instantaneous vertical flux \( J_z(t) \) (bottom curve) near the sidewall. The local heat flux was measured with very good precision, and the measured results are nicely consistent with the theoretical predictions [5] and recent numerical simulations [121]. Recently, by using commercial heat flux sensors, du Puits et al [122] measured the local heat flux simultaneously at the surfaces of the heating and the cooling plates (see [122] for further details).

4. Summary

In this paper, we have reviewed the various experimental techniques that have been used to study closed turbulent flow systems, with the example of two well-known systems: turbulent Taylor–Couette (TC) flow and turbulent Rayleigh–Bénard (RB) convection. In TC flow, we mainly focus on the measurement techniques of the flow structures, the angular velocity of the flow (transport quantity), the radial velocity (wind velocity), the global torque, and the local turbulent angular velocity flux. In RB convection, we focus on the techniques used to measure the flow structures, the temperature and velocity of the fluid, and the global and local heat flux. There have also been many new developments of the measurement techniques for RB and TC systems in recent years, like 3D-PTV for RB [123], the instrumented tracer for...
Lagrangian measurements in RB [124,125], the measurement technique for the effective wind Reynolds number for RB [126], and the high-resolution PIV measurement technique for TC boundary layers [127], etc. These developments are beyond the scope of the present paper. New emerging technologies will certainly provide better experimental techniques for both turbulent TC flow and RB turbulence in future.

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