Role of Substrate Kinetics on Macromolecular Transport and Hybridization in Microchannels

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Abstract

In this paper, a generalized surface-kinetics based model is developed to analytically investigate the influences of the substrate types and the buffer compositions on the macromolecular transport and hybridization in microfluidic channels, under electrokinetic influences. For specific illustration, three typical microchannel substrates, namely Silanized Glass, Polycarbonate and PDMS, are considered, in order to obtain analytical expressions for their zeta potentials as a function of the buffer pH and the substrate compositions. The expressions for the zeta potential are subsequently employed to derive the expressions for respective velocity distributions and hence the corresponding macromolecular concentration profiles under the application of electric fields of identical strengths in all cases. From the analysis presented, the effect of the surface characteristics on the rate of macromolecular hybridization is pinpointed.

1. Introduction

Zeta potential of a solid substrate [1], in contact with a liquid phase, happens to be one of the key parameters that are responsible for the electrokinetic manipulation of biological samples through microfluidic channels. In this respect, it has been well recognized that chemical modification of the solid substrates (and hence, a modification of the zeta potential) can offer with an excellent methodology for controlling the micro-scale transport processes within the fluidic channels. For instance, since bare silica is prone to rapid contamination, silane based modifications of glass substrates (such as coatings with γ-aminopropyltrioxyisilane or ATPS) are common in microfluidics-based DNA assays. Such amine-rich surfaces are ideal for aiding the attachment of DNA strands to the solid substrates [2]. Other than the modification of the common substrate materials, there has also been a recent trend to employ some alternative materials, such as polymers [3], for the fabrication of microfluidic devices, primarily motivated by certain advantageous characteristics of such materials such as thermal stability, permeability to gases, transparency, bio-compatibility, non-toxicity, isotropy and homogeneity.

While a number of experimental studies have been presented in the literature to demonstrate the roles of various substrate types on the electrokinetically-driven microfluidic transport mechanisms, only a little theoretical work has been undertaken to accommodate the explicit influences of the surface kinetics in the pertinent mathematical models of mass, momentum and solutal transport. Aim of the present work is to devise a novel analytical approach for a substrate-kinetics based mathematical modeling of electrokinetically driven momentum and solutal transport in microfluidic channels, in order to obtain closed form expressions depicting the macromolecular transport and hybridization characteristics. The analytical expressions are derived by considering the zeta potential to be a function of the buffer ion concentration and the kinetic constants associated with the surface reactions. The generic problem of macromolecular transport and hybridization considered in this study resembles (although not necessarily restricted to) the electrokinetically driven convective-diffusive transport and hybridization of single-stranded (ss) DNAs with the corresponding
complementary strands (capturing probes) immobilized at specified locations in the bottom wall of the microchannel. For demonstrating the generalized mathematical model, three separate substrate materials, namely, silanized glass, PDMS and polycarbonate, are considered. With surface-kinetics dependent expressions for zeta potentials for each of the substrate types, approximate analytical solutions are subsequently derived for the velocity and the concentration field. These results are also rigorously compared with the corresponding predictions from a full-scale numerical solution of the integrated mathematical model for mass, momentum and solutal transport, so as to assess the effectiveness of the analytical solutions obtained in this study.

2. Mathematical modeling

2.1 Problem description

For mathematical modeling, fluidic transport through a parallel plate microchannel (fig 1) of length L, height 2H and width w (w>>L, w>>2H) is considered.

![Figure 1a: The flow geometry](image)

An electric field is applied along the axis of the channel, which provides the necessary driving force for the electro-osmotic flow. The flow carries the macromolecules, which subsequently get hybridized with their complementary sequences that are immobilized on the surface probes (See Ref 4 for a detailed sketch). A comprehensive description of the mathematical model for mass, momentum, heat and solutal transport, along with the pertinent boundary conditions, is presented in the work of Das et al. and Das and Chakraborty [4, 5], and is not repeated here for the sake of brevity. Regarding the implementation of the boundary conditions, it is important to note here that the surface potential value employed for the solution of the Poisson-Boltzmann equation is a strong function of the reaction kinetics at the liquid-substrate interface, and hence, cannot be imposed arbitrarily, without appealing to the details of the surface reactions. In the present study, accordingly, the surface potential boundary values are derived in a systematic, step-by-step and detailed manner, based on the pertinent reaction kinetics specific to the chosen substrate-solvent pair, as illustrated in table I.

2.2 Approximate solutions to the governing transport equations

Before attempting for an approximate analytical solution of the physico-chemical problem described as above, a comprehensive order of magnitude analysis is performed, so as to assess the relative significance of various terms appearing in the pertinent governing equations (See Refs. 4 and 5) and the resultant equations signifying the balance of the terms of dominant magnitude read:

\[
\frac{d^2\psi}{dy^2} = -\frac{\rho_e}{\varepsilon\varepsilon_0} = -\frac{e(z_+n_+ + z_-n_-)}{\varepsilon\varepsilon_0} \quad \text{(Equation for the EDL potential)} \quad (1)
\]

\[
\mu \frac{d^2u}{dy^2} = -\rho_e E_x \quad \text{(Equation for the velocity field)} \quad (2)
\]

\[
D \frac{d^2C}{dy^2} = -\mu_\varepsilon C \frac{d^2\psi}{dy^2} \quad \text{(Ionic Species Conservation Equation)} \quad (3)
\]

The necessary solutions of the above set of equations under the appropriate boundary conditions (See Refs. 4 and 5) read:

\[
C_i = C_{0,i} \exp \left( -\frac{e\varepsilon}{k_B T} \left( \psi - \psi_0 \right) \right) \quad (4)
\]

\[
\psi = \xi \cosh \left( \frac{\omega (H - y)}{\varepsilon} \right) \quad (5)
\]

\[
u = -\frac{\varepsilon\varepsilon_0 E_x}{\mu} \frac{1 - \psi}{\xi} \quad (6)
\]

2.3 Expressions for the zeta potential

The surface concentration boundary values, as specified by the above equations, can be physically well-posed, only if kinetically-consistent values of the zeta potentials are substituted in the relevant expressions. To obtain the zeta potential values pertaining to three different types of surfaces, we first calculate the surface charge density expressions for each of them as:

Silanized Glass
\[ \sigma_{\text{charge}} = \frac{e\gamma_{PD}}{1 + \frac{[H^+]}{K_a1} + \frac{[H^+]}{K_a2} + K_p [M^{n+}]} \]  
\[ \gamma_{PD} = N_a \left[ \frac{[SiO^-]}{[SiOH]} + \frac{[SiOH]}{[SiOH_2]} \right] \] (7a)

The equilibrium constants are expressed as:

\[ K_1 = \frac{[SiO^-][H^+]}{[SiOH]^2} \]
\[ K_2 = \frac{[SiOH][H^+]}{[SiOH_2]^2} \]
\[ K_{a1} = \frac{[M^{n+}][SiO^-]}{[M^n][SiO^+]}, \quad K_{a2} = \frac{[R-NH_3][H^+]}{[R-NH_3][H^+]} \]
\[ K_N = \frac{M^{n+} \sigma^2}{[M^n][M^{n+}]} \] (7b)

\[ \gamma_{PC} = N_a \left[ \frac{[R-COO^-]}{[M^{n+}+R-COO^-]} \right] \] (9a)

The equilibrium constants are expressed as:

\[ K_{a1} = 10^{-4.5}, \quad K_{a2} = 10^{0.5}, \quad K_N = 10^{-7} \]

\[ \sigma_{\text{charge}} = \frac{e\gamma_{PC}}{1 + \frac{[H^+]}{K_{a3}}} + K_p [M^{n+}] \]

\[ K_p = \frac{[M^{n+}+R-COO^-]}{[R-COO^-]} \]

(9)

From the expression of the charge density \( \sigma_{\text{charge}} \), one can calculate the zeta potential expression for each surface from the equation:

\[ \zeta = \frac{F\lambda k_B}{e\varepsilon_0 R} \sigma_{\text{charge}} \] (10)

Obtaining closed-form expressions of the surface charge density \( \sigma_{\text{charge}} \), and hence of the zeta potential \( \zeta \) gets further complicated by the fact that \( \sigma_{\text{charge}} \) itself can be expressed in terms of the hydrogen ion and the metallic ion concentrations at the channel surface again depend on the local zeta potential (eqs. 4, 5).

2.3.1 Zeta potential for silanized glass

The zeta potential for the Silanized glass, as an outcome of the above analysis, can be obtained as a root of the following equation:

\[ M_{D} C_{i} \zeta^3 + (B_{i} M_{i} D_{i} + M_{i} C_{i} - D_{i} A_{i}) \zeta^2 \]

(11)

Here \( K_{a3} = 10^{-3.92}, \quad K_p = 10^{-7} \)

Order of magnitude analysis of the terms show that for \( |\zeta| < 1 \text{ V} \), terms with higher powers of \( \zeta \) are small enough so that eq. (11) reduces to...
\[
\left( M_1 C_4 + M_1 C_1 B_2 - C_1 A_2 - D_1 D_2 \right) \zeta^2 + \left( M_1 B_4 - B_1 A_2 - A_1 B_2 - C_1 D_2 \right) \zeta \]
\[
- \left( A_1 C_2 + B_2 D_1 \right) = 0
\]
so that
\[
\zeta = \frac{-M_1 B_4 - B_1 A_2 - A_1 B_2 - C_1 D_2 \pm \sqrt{\left( M_1 B_4 - B_1 A_2 - A_1 B_2 - C_1 D_2 \right)^2 + 4 M_1 C_4 + M_1 C_2 B_2 - C_1 A_2 - D_1 D_1 \left( A_1 C_2 + B_2 D_1 \right)}}{2 M_1 C_4 + M_1 C_2 B_2 - C_1 A_2 - D_1 D_1}
\]
(13)
The other root of eq. (12) produces physically unrealistic result and is accordingly neglected.

2.3.2 Zeta potential for PDMS
The polynomial involving the zeta potential for the PDMS is given as
\[
MC_5 \zeta^2 + \left( M_1 B_4 - C_4 \right) \zeta^2 + \left( M_1 A_4 - B_4 \right) \zeta - A_4 = 0
\]
(14)
where
\[
A_4 = e \gamma_{PD} \left[ 1 - \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1} K_{a_2}} \right] B_4 \frac{\varepsilon_{fPD}}{k_BT} \left( \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1} K_{a_2}} \right)
\]
\[
C_3 = -\frac{e^3 \gamma_{PD}}{k_BT} \left( \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1} K_{a_2}} \right)
\]
\[
A_4 = 1 + \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1}} + \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_2}} + K_N \left[ M^+ \right]^0
\]
\[
B_4 = -\frac{e}{k_BT} \left( \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1}} - 2 \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1} K_{a_2}} + K_N \left[ M^+ \right]^0 \right)
\]
\[
C_4 = -\frac{\left( \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_1} K_{a_2}} \right)^2}{K_{a_1} K_{a_2}}
\]
Similar order of magnitude analysis reduces eq. (14) to:
\[
\left( M_1 B_4 - C_4 \right) \zeta^2 + \left( M_1 A_4 - B_4 \right) \zeta - A_4 = 0
\]
(15)
so that,
\[
\zeta = \frac{-\left( M_1 A_4 - B_4 \right) \pm \sqrt{\left( M_1 A_4 - B_4 \right)^2 + 4 M_1 B_4 - C_4 A_4}}{2 M_1 B_4 - C_4}
\]
(16)
The other root of eq. (15) produces physically unrealistic result and is accordingly neglected.

2.3.3 Zeta potential for polycarbonate
The polynomial expression for the zeta potential of polycarbonate is given as
\[
M_1 C_5 \zeta^2 + M_1 B_5 \zeta - A_5 = 0
\]
(17)
where
\[
A_5 = e \gamma_{PC}
\]
\[
B_5 = 1 + \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_3}} + K_P \left[ M^+ \right]^0
\]
\[
C_5 = -\frac{e}{k_BT} \left( \frac{\left( H^+ \right)^{\frac{1}{2}}}{K_{a_3}} + K_P \left[ M^+ \right]^0 \right)
\]
The final expression for the zeta potential for the polycarbonate, accordingly, is given as
\[
\zeta = \frac{-M_1 B_5 - \sqrt{(M_1 B_5)^2 + 4 M_1 C_5 A_5}}{2 M_1 C_5}
\]
(18)
The other root of eq. (17) produces physically unrealistic result and is accordingly neglected.

2.4 Solution for the macromolecular concentration distribution
The velocity field, obtained as above, can be substituted in the species conservation equation for the macromolecules, to obtain their concentration distribution as a function of position and time. For obtaining an analytical solution for the same, following assumptions are made:

(i) The non-specific adsorption at the channel bottom wall is neglected, so that the hybridization kinetics get simplified as
\[
\frac{\partial C_H}{\partial t} = k_3^0 \left( C_{H,max} - C_H \right) C_{film} \text{ and } k_3^1 \text{ are the kinetic constants for the association and the dissociation of the target macromolecules with the immobilized complementary macromolecules. In eq. (21), } C_{film}\text{ refers to the solution phase concentration of the target macromolecules at the surface film, and } C_{H,max}\text{ is the maximum concentration possible for the hybridized macromolecules.}
\]
(ii) A linear axial concentration gradient is imposed across the microchannel, i.e.,
\[
\frac{\partial C}{\partial x} = \text{constant} = M_C
\]
The final form of the macromolecular species conservation equation, coupled with the appropriate initial and boundary conditions, can be summarized as follows:
\[
\frac{\partial C}{\partial t} + (u + \mu_x E_x) M_c = D \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial}{\partial y} \left( \mu_y C \frac{\partial y}{\partial y} \right) \right) \tag{20}
\]
\[C(0, y) = 0 \quad \text{for } 0 \leq y \leq 2H \tag{20a}\]
\[
D \frac{\partial C(t, 0)}{\partial y} = -\frac{\partial C}{\partial t} \quad \text{for } t > 0 \tag{20b}
\]
\[
\frac{\partial C(t, H)}{\partial y} = 0 \quad \text{for } t > 0 \tag{20c}
\]
\[
\frac{\partial C(t, 2H)}{\partial y} = 0 \quad \text{for } t > 0 \tag{20d}
\]

An expression for the macromolecular concentration field can finally be obtained by solving the above equation, as

\[
C = \sum \left[ \cos \frac{n \pi y}{2H} \frac{S_y - \exp(-F_y) + \frac{S_y}{R} \left( \sin \frac{n \pi y}{2H} \right) \exp(-R_y)}{\exp(-F_y) + \frac{S_y}{R} \left( \sin \frac{n \pi y}{2H} \right) \exp(-R_y)} \right]
\]

\[
- \frac{E_0 \exp(-F_0)}{2D} \left( \frac{2h^2 - y^2}{2} \right) \tag{21}
\]

Various parameters appearing in eq. (21) are as follows:
\[
R_1 = \frac{D n^2 \pi^2}{4H^2} - \mu_y \frac{\zeta \eta}{H} \tanh \left( \omega H \right) \tag{21a}
\]
\[
E_B = k_1^1 C_{H, \text{max}} C_{\text{film}} \tag{21b}
\]
\[
F_B = k_3^{-1} + k_3 C_{\text{film}} \tag{21c}
\]
\[
S_1 = \left[ \frac{L_4 + \mu_y \exp(-\omega H)}{2D} \right] \tag{21d}
\]
\[
S_2 = \left[ \frac{L_4 + \mu_y \exp(-\omega H)}{2D} \right] \tag{21e}
\]
\[
S_3 = \left[ \frac{L_4 + \mu_y \exp(-\omega H)}{2D} \right] \tag{21f}
\]
\[
S_4 = \left[ \frac{L_4 + \mu_y \exp(-\omega H)}{2D} \right] \tag{21g}
\]
\[
L_1 = \left( \frac{2H}{n \pi} \right)^2 \cos \left( \frac{n \pi}{2} \right) - 1 \tag{21h}
\]
\[
L_2 = -\frac{4H}{n \pi} \cos \left( \frac{n \pi}{2} \right) \tag{21i}
\]

\[
L_3 = -\frac{16H^3}{(n \pi)^2} \cos \left( \frac{n \pi}{2} \right) \tag{21j}
\]
\[
L_4 = \frac{8H^3}{n \pi} \cos \left( \frac{n \pi}{2} \right) + \frac{16H^3}{n \pi} \cos \left( \frac{n \pi}{2} \right) - 1 \tag{21k}
\]
\[
L_5 = 0 \tag{21l}
\]
\[
L_6 = -\frac{2H}{n \pi} \cos \left( \frac{n \pi}{2} \right) - 1 \tag{21m}
\]
\[
L_7 = \frac{4H^3}{4H^3 + n \pi \pi} \left( \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \right) \tag{21n}
\]
\[
L_8 = -\frac{2H}{n \pi} \left( \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \right) \tag{21o}
\]
\[
L_9 = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21p}
\]
\[
L_{10} = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21q}
\]
\[
L_{11} = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21r}
\]
\[
L_{12} = 2H \pi \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21s}
\]
\[
L_{13} = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21t}
\]
\[
L_{14} = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21u}
\]
\[
L_{15} = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21v}
\]
\[
L_{16} = \frac{8H}{4H^3 + n \pi \pi} \exp \left( 2\omega H \right) \cos \left( n \pi \right) - 1 \tag{21w}
\]
The concentration field, as obtained from equation (21), can be employed to derive the temporal variation of the rate of macromolecular hybridization at a time \( t=5 \) M.

\[
\frac{dC_{\text{d+t}}}{dt} = \frac{d}{dt} \left( \frac{k_{z,\text{d+t}} C_{\text{n,\text{d+t}}} f_{\text{d+t}}(t)}{g_{\text{d+t}}(t)} \right) \tag{22}
\]

where \( f_{\text{d+t}}(t) \) and \( g_{\text{d+t}}(t) \) are described as follows:

\[
g_{\text{d+t}}(t) = \exp \left( \int_{0}^{1} [k_{z,\text{d+t}} + k_{\text{a,\text{d+t}}} C(0, t)] dt \right) \tag{22a}
\]

\[
f_{\text{d+t}}(t) = \int_{0}^{1} g_{\text{d+t}}(t) C(0, t) dt \tag{22b}
\]

3. Results and Discussions

The necessary parameters used for the present problem are identical to that of Refs 4 and 5. Fig. 2 depicts the variation of zeta potential as a function of the pH value of the inlet buffer, for the three different substrate materials considered in this study.

**Figure 2**: Variation of the zeta potential with the pH of the inlet buffer for different substrates, corresponding to an ionic concentration of 0.001 M.

The range of pH values (3<\( \text{pH} <5 \)) considered for obtaining these plots is taken to be such that the p.z.c values corresponding to all the chosen substrate materials fall beyond this range. While p.z.c values for PDMS and polycarbonate are less than the lower limit of this range, the p.z.c value for silanized glass is greater than the corresponding upper limit. Thus, the zeta potential values of PDMS and polycarbonate are negative over the entire range, whereas that of the silanized glass is positive. In this regard, it is important to observe from fig. 2 that the magnitude of zeta potential in case of silanized glass is substantially higher than that of the other two substrate types, over the range of pH considered. This clearly justifies a strategic employment of silane-based microchannel substrates, to be exposed to the bulk liquid flow, so that the electrokinetic flow strength can substantially be enhanced.

Fig. 3 depicts the variations in the magnitude of the axial velocity component as a function of the distance from the channel walls, over half width of the channel, for inlet buffer pH=4. Since the magnitude of silanized glass is greater than that of polycarbonate and that of PDMS is the lowest for the three substrates for pH=4, the electroosmotic flow strength is also found to be the highest for silanized glass and the lowest for the PDMS substrate. It is important to note here that similar qualitative trends in the velocity profiles are observed for the other values of pH as well, which are not detailed here for the sake of brevity. The only distinctive feature to emphasize in this respect is the difference in the magnitudes of the bulk velocity between the silanized glass channel and the other two substrates is much larger for pH values less than 4, whereas for pH values greater than 4, this difference becomes substantially smaller.

**Figure 3**: Analytical and numerical results of the variation of the x-component of the velocity \( (u) \) with normalized channel height \((y/H)\) for the channel bottom half plane for microchannels for an inlet buffer of \( \text{pH} = 4 \). Only magnitudes of the velocities are represented in the figure.

Fig. 4 depicts the variations in the macromolecular film concentrations at a time \( t=5 \)
sec, as a function of the inlet buffer pH in the range of 3 to 5. It is revealed that the film concentration increases with increase in pH for polycarbonate and PDMS substrates, whereas the same decreases with increase in pH for silanized glass. This is consistent with the fact the magnitude of the zeta potential and hence the average flow strength increase with increase in pH for the former two, while the reverse trend is observed for silanized glass (see fig. 2).

Figure 4: Numerical and analytical results for the variation of the film concentration ($C_{\text{film}}$) of the macromolecules with the pH of the inlet buffer for the microchannels of different substrates at time $t=5$ sec.

Fig. 5 represents the variations in the rate of macromolecular hybridization at the capturing probes as a function of time, for inlet buffer pH=4. In general, the rate of hybridization decreases with increase in time, for all cases. This can be attributed to the fact that with higher concentration of hybridized targets, the rate of dissociation of the hybridized macromolecules also increases, so that the rate of hybridization eventually arrives at a condition of saturation or local equilibrium, at which the rates of association and dissociation become competitive. Here again, the analytical solutions are found to slightly under-predict the actual hybridization rates, primarily because of the reasons mentioned as earlier. The under-prediction is further aggravated by the fact that the numerical solutions consider an initial non-specific adsorption rate in the macromolecular reaction and hybridization characteristics, whereas the same is neglected for deriving the analytical solutions. Effects of this simplification become more prominent for higher values of the film concentration. As a result, the differences between the analytically and numerically predicted estimates of the rate of hybridization are more prominent for the silanized substrate, as compared to the other two substrate types. In all cases, however, a reasonably good qualitative agreement between the analytical and the numerical predictions of the rate of hybridization can be observed.

Fig. 6 possibly represents the most significant technological outcome in the perspective of the present study, by comparing the macromolecular hybridization times (i.e., the time taken to achieve saturation kinetics in the macromolecular hybridization reaction) for the different substrate types, as a function of the inlet buffer pH. While the hybridization time decreases with increase in pH for polycarbonate and PDMS substrates, a reverse trend is observed for the silanized glass substrates. This can be attributed to the characteristic variation of the magnitude of the zeta potential with increases in pH for the different substrate types over the ranges of pH covered, as detailed earlier. A general conclusion from all cases is the fact that the required hybridization time is the shortest for the pH value that is deviated the most from the p.z.c value of the corresponding substrate type. In all cases, the analytical solutions slightly underpredict the hybridization time, for reasons mentioned as earlier. With this systematic trend, the present analytical solutions can clearly be deployed as conservative estimates for the design of microfluidics based optimized active macromolecular hybridization systems.

Figure 5: Numerical and analytical results for the temporal variation of the rate of macromolecular hybridization ($\frac{\partial C_H}{\partial t}$) for different substrates, corresponding to a buffer pH of 4.
It can be mentioned in this context, that the present study is well able to predict the hybridization characteristics for the case with fixed wall zeta potential for a given substrate, provided that value of the zeta potential is within the range of the zeta potential values covered in the operating pH range for the present problem. In most of the existing literatures three parameters namely the substrate type, the buffer pH and the constant wall zeta potential are independently stated. This is not quite correct, as fixing up of any two parameters automatically fixes the third one. Thus for a given substrate and a given pH, one always encounters a fixed value of the zeta potential, that remains constant independent of the channel axial location. This becomes possible due to the fact that the hydrogen and the metallic ion concentrations that regulate the zeta potential show negligibly small variation with axial direction (already illustrated in the scaling analysis presented earlier).

4. Conclusions

In this paper, a novel analytical model has been developed, which employs the detailed surface kinetics to obtain closed-form expressions of the zeta potential values for the standard microfluidic substrates such as silanized glass, polycarbonate and the PDMS. These expressions for the zeta potentials have subsequently been employed to analytically solve the coupled mass, momentum and solutal transport equations, in order to predict the macromolecular transport and hybridization characteristics associated with the microfluidic channels. The analytical solutions have been comprehensively compared with the corresponding full-scale numerical predictions, and a favourable matching has been observed for all cases. It has been suggested that with systematic trends in predictive capabilities, the present analytical solutions can safely be employed as conservative system-design estimates, without going for time-intensive numerical computations.

References