Augmentation of macromolecular adsorption rates through transverse electric fields generated across patterned walls of a microfluidic channel

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The present study aims to establish a theoretical proposition that the rate of macromolecular adsorption can be augmented with application of transverse electric fields across patterned walls of a microfluidic channel. First, an approximate fully developed velocity profile is derived, which is subsequently utilized to solve the species conservation equation pertaining to a combined advection-diffusion transport. Closed form solutions for the concentration field are subsequently obtained, in consistency with the typical second order kinetics of macromolecular adsorption. It is suggested that the transverse electric field and pattern angles can be regulated as independent parameters to maximize the rate of macromolecular adsorption under the prevailing conditions.


I. INTRODUCTION

Control of fluid flow through microchannels is of utmost practical significance, with applications ranging from analytical and synthetic chemistry to DNA microarray technologies. Although a pressure-driven fluid flow could ideally be one of the obvious means to achieve this goal, it offers certain inconveniences, such as giving rise to the phenomenon of Taylor’s dispersion on account of a parabolic nature of the velocity profile. Not only that, a huge pumping power is required to generate the pressure gradient, because of the prevailing scaling laws in the microdomain. On the contrary, electro-osmosis has turned out to be a more convenient way to establish nearly plug-shaped velocity profiles in microchannels. This has lead to reduced dispersion effects in various microfluidic devices, resulting in a limited dilution of samples, and consequently, an improved processability for separation purposes.

Commonly, an external axial electric field is applied to achieve the purpose of separation and migration of different types of analytes in a microfluidic channel. However, for situations which demand the movability of analytes (especially biological macromolecules) in a transverse direction, application of transverse potential gradients, in addition to an axial one, may turn out to be very useful. This is primarily because of the fact that a transverse electric field may offer, with an additional controlling parameter, to enhance the rate of macromolecular transport and hybridization and improve the stringency of nucleic acid interactions. Very often, such transverse potential gradients are functions of the buffer properties and channel geometries. Of particular interests may be channels with patterned surfaces. Such arrangements, in effect, can generate additional pressure gradients along the axial direction, and thereby hold the capability of augmenting the rate of macromolecular transport beyond the limits imposed by axial electric fields alone.

The primary focus of the present study is to explore the possibilities of augmenting the rate of adsorption of uncharged biological macromolecules on the bottom surface of microchannels, by means of transverse electric fields generated across patterned walls of the channel, from a theoretical perspective. A comprehensive mathematical model is developed and solved analytically, to depict the inter-relationship between imposed axial and transverse electric fields, patterns on the channel wall, and the rate of adsorption, by assuming second order adsorption kinetics. This concept and theory, in the future, can be exploited to design and fabricate microfluidic channels, ensuring a faster rate of adsorption of biological macromolecules in comparison to what could be achieved within the current technological constraints.

II. PROBLEM FORMULATION

A. Solution of the velocity field

We consider a microchannel of length $L_0$, with a typical Hele-Shaw quasiplanar geometry, as shown Fig. 1(a). The channel has a large extent in a direction perpendicular to the longitudinal plane of flow. The two walls of the channel [Fig. 1(b)] contain periodic patterns with principal axes $(x_1, y_1)$, where the axis “$x_1$” is perpendicular to the direction of the alignment of the patterns, and the axis “$y_1$” is aligned along the direction of the pattern. The axis $x_1$ makes an angle $\theta$ with the $x$ axis, which is oriented along the channel length. The presence of such patterns effectively ensures that the two-dimensional velocity fields are related to the two-dimensional currents on a local scale. The physical situation prevailing under these conditions is somewhat analogous to the case in which a parallel flow through a microchannel is disturbed by the influence of its wavy walls. Under these circumstances, by introducing a normal mode assumption for the perturbed stream function $[\phi(x, y)]$ in the fluid flow equations as

$$\hat{\phi}(x, y) = \phi(y) \exp(\alpha x / H),$$

(1)

(where $\alpha$ is the nondimensional wave number given as $\alpha = 2\pi H / \lambda$, $H$ being the channel height, and $\lambda$ being the wave-
Accordingly, no-slip velocity condition prevails at the solid boundaries. Its derivatives
respectively, and where
is the applied electric field in the axial direction. To solve Eq. (2), appropriate boundary conditions are imposed on \( \phi \) and its derivatives (with respect to \( y \)), so as to ensure that a no-slip velocity condition prevails at the solid boundaries. Accordingly,

\[ \phi = \delta_\phi \quad \text{at} \quad y = H + n, \]  
\[ \phi' = 0 \quad \text{at} \quad y = H + n, \]  
\[ \phi = \delta_\phi \quad \text{at} \quad y = n, \]

Here, \( n = A \cos(\alpha x/H) \) represents the waviness in the walls, where \( A \) is the amplitude of the undulations of the channel surface, and the primes denote differentiation with respect to \( y \). The constant value of \( \phi \) at the channel walls, denoted by \( \delta_\phi \), can be taken as \( \delta_\phi = \lambda_{DB} \), where \( \lambda_{DB} \) is the Debye length which characterizes the thickness of the electric double layer that forms near the channel walls, due to a preferential distribution of the ionic charges present in the solution, in response to the ionization of the channel walls.

For an approximate solution of Eq. (2), one may seek an asymptotic polynomial expansion of the variable \( \phi \), and obtain the following variations, in the limit of small \( n \):

\[ \phi = \delta_\phi + \frac{H^2 \delta_\phi (72 - A^2)}{2A'H^2 - 24} y^2 \]
\[ + \frac{H \delta_\phi (A'H^2 - 3A^2 y^2 - 216)}{9(A'H^2 - 12)} y^3 \]
\[ + \left[ \frac{H^2 \delta_\phi A'H^3 - A^2 \delta_\phi}{24 (A'H^2 - 12)} \right] y^4, \]  
where

\[ A'_1 = 2 \alpha^2 - \frac{u_{HS}}{\sinh(\omega H)} \left[ \sin(\omega H) + \omega + \omega^3 H \right], \]
\[ A'_2 = \frac{H^2 \alpha d \rho dp}{\mu^2} \frac{u_{HS}}{\sinh(\omega H)} \left[ \sinh(\omega H) + \omega \right], \]  

Equation (5) can be employed to obtain the perturbed \( x \) component of velocity, at a given axial location \( x = x_0 \), as

\[ u_{\text{per}} = \text{Re}[ \phi' \exp(i \alpha x_0/H) ], \]  

which ultimately leads to the following velocity profile, at an axial location \( x = x_0 \):

\[ u(y) = U_0(y) + u_{\text{per}} = U_0(y) + \text{Re}[ \phi' \exp(i \alpha x_0/H) ] \]
\[ = -\frac{1}{2 \mu} \frac{d}{dx} \frac{E_y}{\mu} \left[ \sinh(\omega H) - \sin(\omega H) \right] \]
\[ - \left[ 2 K_1 y + 3 K_2 y^3 + K_3 y \right] \sin(\alpha x_0/H), \]  

where

\[ K_1 = \frac{36 H^2 \delta_\phi}{K_4 H^2 - 24}, \]
\[ K_2 = \frac{H^2 \delta_\phi}{K_4 H^2 - 24} \left[ \frac{H \rho \alpha dp}{\mu^2} \frac{u_{HS}}{\sinh(\omega H)} \right. \]
\[ \times \left. \left[ \frac{\omega^3 \rho \alpha}{\mu} + \sinh(\omega H) \frac{\rho \alpha}{\mu^3} \right] + \frac{\omega \rho \alpha^3}{\mu^2} + \frac{\omega^3 \rho \alpha^3}{6 \mu} \right], \]  
\[ \text{(7a)} \]
\[ \text{(7b)} \]
Again the elements of the matrix responsible for the flow due to the pressure gradient, whereas the conduction matrix \( S \) decides the electric field. The matrix \( M \) governs the electrohydrodynamic coupling (the electrokinetic effects).

To obtain the additional equivalent pressure gradient \( \nabla p_e \) as functions of the hydrodynamic and the electric current, defined by Eq. (8b), one needs to obtain the elements of matrices \( k \) and \( m \). These parameters, in turn, depend on the corresponding values along the direction of the pattern and perpendicular to the direction of the pattern. With the subscript “1” denoting values perpendicular to the direction of the pattern (i.e., along the \( x1 \) axis), and the subscript “2” denoting values along the direction of the pattern (i.e., along the \( y1 \) axis), one may write

\[
\begin{bmatrix}
    f_{xx} & f_{xy} \\
    f_{yx} & f_{yy}
\end{bmatrix} = \begin{bmatrix}
    f_2 \cos^2 \theta + f_1 \sin^2 \theta & (f_1 - f_2) \sin \theta \cos \theta \\
    (f_1 - f_2) \sin \theta \cos \theta & f_1 \cos^2 \theta + f_2 \sin^2 \theta
\end{bmatrix},
\]

(10)

where \( \theta \) is the angle between the \( x \) and \( x1 \) axes, and \( f \) is a generic representation of either \( k, s, \) or \( m \). Further, it can be noted that

\[
k_1 = 12 \mu \left( \frac{1}{h^3} \right),
\]

(11a)

\[
k_2 = 12 \mu (h^3)^{-1},
\]

(11b)

where \( h \) is a variable denoting the microchannel height, with an average value of \( H \). For a given surface geometry, these can be simplified as

\[
k_1 = \frac{12 \mu}{H^3} (1 + 3 \tilde{A}^2),
\]

(11c)

\[
k_2 = \frac{12 \mu}{H^3} \left( 1 - \frac{3}{2} \tilde{A}^2 \right),
\]

(11d)

which leads to the following expression for the matrix \( k \):

\[
\begin{bmatrix}
    k_{xx} & k_{xy} \\
    k_{yx} & k_{yy}
\end{bmatrix} = \begin{bmatrix}
    \frac{12 \mu}{H^3} \left[ 1 + 3 \tilde{A}^2 \left( \sin^2 \theta - \frac{\cos^2 \theta}{2} \right) \right] & \frac{54 \mu \tilde{A}^2}{H^3} \cos \theta \sin \theta \\
    \frac{54 \mu \tilde{A}^2}{H^3} \cos \theta \sin \theta & \frac{12 \mu}{H^3} \left[ 1 + 3 \tilde{A}^2 \left( \cos^2 \theta - \frac{\sin^2 \theta}{2} \right) \right]
\end{bmatrix}.
\]

(12)
\( m_1 = \frac{6\mu}{\gamma_{el}} \left( \sigma_1 + \sigma_2 \right) \frac{h}{h^3}, \)  
(13a)

and

\( m_2 = \frac{6\mu \left( \left( \sigma_1 + \sigma_2 \right) h \right)}{\gamma_{el} \left( h/h^3 \right)}, \)  
(13b)

where \( \gamma_{el} \) is the electrical conductivity of the solution, \( \sigma_1 \) is the net charge density on the bottom plate, and \( \sigma_2 \) is the net charge density on the top plate. For the patterned surfaces, the following expressions can be obtained from Eqs. (13a) and (13b):

\[ m_1 = \frac{12\mu\sigma_0}{\gamma_{el}H^3} \left( 1 - \frac{3}{2} A \frac{\delta\sigma}{\sigma_0} \cos\varphi + 3A^2 \right), \]  
(13c)

\[ m_2 = \frac{12\mu\sigma_0}{\gamma_{el}H^3} \left( 1 + \frac{1}{2} A \frac{\delta\sigma}{\sigma_0} \cos\varphi - \frac{3}{2} A^2 \right), \]  
(13d)

where \( \sigma_0 \) is the average charge density. In Eqs. (13c) and (13d), the terms \( \delta\sigma \) and \( \cos\varphi \) take care of the surface charge modulation due to the variation in the surface geometry, and are related to the charge density of the two plates by the relation

\[ \sigma_1 + \sigma_2 = 2[\sigma_0 + \delta\sigma \cos(\alpha x + \varphi)]. \]  
(13e)

The above expressions lead to the following form of the \( m \) matrix:

\[
\begin{bmatrix}
m_{xx} & m_{xy} \\
m_{xy} & m_{yy}
\end{bmatrix}
= \frac{12\mu\sigma_0}{\gamma_{el}H^3} \left[ \frac{1 + A \frac{\delta\sigma}{\sigma_0} \cos\varphi - 2A \frac{\delta\sigma}{\sigma_0} \cos\varphi \sin^2 \theta + 3A^2 \sin^2 \theta - \frac{3A^2}{2} \cos^2 \theta}{2} \right] \\
12\mu\sigma_0 \left[ \frac{9A^2}{2} - 2A \frac{\delta\sigma}{\sigma_0} \cos\varphi \right] \sin \theta \cos \theta
\]

(14)

From Eq. (12), we obtain \( k_{xx} \approx k_{xy} \), by noting that \( A \) is the amplitude of the surface modulation, which is of the order of the channel height, i.e., of the order of a few microns. Analogously, it follows from Eq. (14) that \( m_{xx} \approx m_{xy} \). Hence, under the assumption\(^7\) that \( J_{el,v} \) is smaller than \( J_{el,x} \) and \( J_y \) is smaller than \( J_x \), from Eq. (9b) one obtains

\[
\frac{\partial p}{\partial x} \approx \frac{\partial p}{\partial y}.
\]

Finally, to evaluate components of the \( s \) matrix, we note that\(^7\)

\[ s_1 = \frac{1}{\gamma_{el}} \left( \frac{1}{h} \right), \]  
(15a)

\[ s_2 = \frac{1}{\gamma_{el}} (h)^{-1}. \]  
(15b)

For the given surface geometry, the above can be simplified as\(^7\)

\[ s_1 = \frac{1}{\gamma_{el}H} \left( 1 + \frac{A^2}{2} \right), \]  
(15c)

\[ s_2 = \frac{1}{\gamma_{el}H}. \]  
(15d)

This leads to the following final form of the \( s \) matrix:

\[
\begin{bmatrix}
s_{xx} & s_{xy} \\
s_{xy} & s_{yy}
\end{bmatrix}
= \frac{1}{\gamma_{el}H} \left[ \frac{1 + A^2 \sin^2 \theta}{2} \right] \\
\frac{A^2}{2 \gamma_{el}H} \sin \theta \cos \theta
\]

(16)

Finally, noting that \( J_{el,x} = H(\gamma_{el}E_x) \), from Eq. (8) one obtains

\[
\frac{d\rho}{dx} = m_{xx} H \gamma_{el} E_x + k_{xx}
\]

\[
\times \frac{m_{xx} (E_x - s_x H \gamma_{el} E_x - s_y H \gamma_{el} E_y) - m_{xy} (E_y - s_y H \gamma_{el} E_y - s_x H \gamma_{el} E_x)}{m_{xx} m_{xy} - m_{xy} m_{xx}}
\]  
(17)

This pressure gradient [Eq. (17)] augments the externally imposed pressure gradient, which is mainly axial in nature, while the transverse pressure gradient remains rather insignificant. This essentially implies that a fully developed velocity profile may be considered for an approximate analysis. Accordingly, for the case of an applied transverse electric field across the micropatterned walls, one can employ a velocity profile of the same form as that of Eq. (7), with the pressure gradient term modified to incorporate the effects of an excess equivalent pressure gradient over and above the imposed one. A major implication of Eq. (16) lies in the fact that by applying an electric field in the transverse direction,
one can augment the rate of microfluidic transport, through an enhancement in effective axial pressure gradients. However, this augmentation in axial flow strength may not be achievable through the application of a transverse electric field alone, until and unless the wall pattern angle is set to be nonzero (so as to ensure that $m_{xy}$ and $s_{xy}$ are both nonzero, in turn), which clearly exhibits the utility of surface patterning to enhance the rate of fluid flow through the microchannel.

**B. Solution of the concentration field**

The governing equation for species conservation, in a dimensional form, assuming that the species are uncharged, is given as

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial x} \left( D_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_y \frac{\partial C}{\partial y} \right),$$

(18)

where $D_x$ and $D_y$ are the diffusion coefficients along $x$ and $y$ directions, respectively. For solution of Eq. (18), we assume the fluid to be of constant physical properties, and a linear bulk concentration gradient $[(\partial C/\partial x)=M$, say] is imposed along the microchannel axis. However, due to macromolecular adsorption at the microchannel walls, local concentration variations may occur along the $y$ direction. A transience in surface concentration may also feature, primarily attributable to time-dependent surface reaction kinetics. It is also assumed here that the rate at which the desorption of macromolecules (i.e., removal of the adsorbed macromolecules from the surface back to the bulk solution) takes place is much slower as compared to the rate adsorption. This renders the concentration field, $C(t,y)$, being subjected to the following initial and boundary conditions: initial condition

$$C(0,y) = 0 \quad \text{for } 0 \leq y \leq H,$$

(18a)

boundary conditions

$$\frac{\partial C(t,H+n)}{\partial y} = 0 \quad \text{for } t > 0,$$

(18b)

$$D_y \frac{\partial C(t,n)}{\partial y} = - \frac{\partial C_H}{\partial t} \quad \text{for } t > 0,$$

(18c)

where $n=A \cos(\alpha x_0/H)$, and $C_H$ is the instantaneous surface phase concentration of the adsorbed macromolecules. The relationship between $C_H$ and the concentration in the bulk is given as

$$B_2 = \frac{2H}{(r \pi)^2} \{MR_2 \cos(r \pi) - [1 - \cos(r \pi)](MR_1 + 2nMR_2)\} - e_\xi E_\omega M$$

$$\times \cosh(\omega m) - \cosh[\omega(H+n)]\cos(r \pi) + \cosh(\omega m)\cos(r \pi) - \cosh[\omega(H-n)] \mu_\omega \sinh(\omega H) \left[1 + (r \pi/\omega H)^2\right]$$

$$+ 3MR_3 \left[\cos(r \pi) - 1\right] \left(H/\pi\right)^2 \left[(H+n)^2 + 2\left(H/\pi\right)^2\right].$$

In the above expressions,

$$\frac{\partial C_H}{\partial t} = k_d(C_{H,\text{max}} - C_{\text{film}}) - k_d C_H,$$

(19)

where $C_{\text{film}}$ is the bulk concentration in the surface film, $k_d$ is the kinetic constant for adsorption of the target with the channel surface, and $k_d$ is the kinetic dissociation constant for desorption. In Eq. (19), $C_{\text{film}}$ refers to the solution phase concentration of the macromolecules at the surface film, and $C_{H,\text{max}}$ is the maximum concentration possible for the adsorbed macromolecules. The first term in right-hand side (RHS) of Eq. (19) represents second order kinetics of macromolecular adsorption, considering that the rate of adsorption depends on the concentration of the nonadsorbed macromolecules close to channel surface as well as the concentration of the already adsorbed macromolecules. In contrast, the kinetics of desorption depends only on the concentration of the already adsorbed macromolecules, and approximately follows first order reaction kinetics [second term in RHS of Eq. (19)].

Equation (18), coupled with initial and boundary conditions specified by Eqs. (18a)–(18c) and (19), lead to the following closed form solution for the concentration field:

$$C(t,y) = \sum_{r=1}^{\infty} w_r(t) \cos \frac{r \pi (y-n)}{H} \frac{e^{-M_1 t}}{2H} \left[2(H+n) - y^2\right].$$

(20)

Various parameters appearing in Eq. (13) are defined as follows:

$$E = k_a C_{H,\text{max}} C_{\text{film}},$$

(20a)

$$F = k_d + k_d C_{\text{film}},$$

(20b)

$$w_r(t) = - \frac{2EH}{D(r \pi)} e^{-M_1 t} + B_3 \frac{1 - e^{-M_1 t}}{M_1 - F(e^{-M_1 t} - e^{-M_1 t})},$$

(20c)

where

$$M_1 = \left(\frac{r \pi}{H}\right)^2,$$

$$A_2 = - \frac{2H}{(r \pi)^2} \frac{1}{D},$$

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\[ R_1 = \frac{H^2 dp}{2\mu} + 2K_1 \cos(\alpha_0/H) - 2K_2 \sin(\alpha_0/H), \]
\[ R_2 = \frac{H^2 dp}{2\mu} + 3K_4 \cos(\alpha_0/H) - 3K_6 \sin(\alpha_0/H), \]
\[ R_3 = K_7 \cos(\alpha_0/H) + K_8 \sin(\alpha_0/H), \]
where \( dp/dx = (dp/dx)_{ext} + (dp/dx)_A \), with \( (dp/dx)_A \) being given by Eq. (17).

### C. Calculation of the concentration of adsorbed macromolecules

It is important to mention at this point that the aim of the present study is to use the close-formed expressions depicted by Eq. (20) for obtaining the variation of the concentration of the adsorbed macromolecules \( (C_H) \) as a function of time. For this purpose, we solve an equivalent ordinary differential form of Eq. (19) and obtain
\[ C_H = \frac{k_c C_{H, max}(t)}{g(t)}, \]
where \( f(t) \) and \( g(t) \) are described as functions of \( t \) as
\[ g(t) = e^{\int_0^t (k_e + k_s C(0,t))dt}, \]
\[ f(t) = \int_0^t g(t) C(0,t)dt. \]

### III. RESULTS AND DISCUSSIONS

Figures 2(a) and 2(b) demonstrate the temporal variation of concentration of the adsorbed macromolecules, corresponding to transverse and longitudinal electric fields of differing orders of magnitude, for a specific pattern angle. The relevant values of physical properties and problem data are enlisted in Table I. From Fig. 2(a), it is revealed that for a low value of the axial electric field \( (E_a = -10 \text{ V/m}) \), stronger transverse electric fields lead to a significant enhancement in the rate of macromolecular adsorption. This is primarily attributable to an enhanced rate of accumulation of the macromolecules close to the channel surface at a given instant of time. However, more interestingly, in case of a strong axial potential gradient \( (E_a = -10^6 \text{ V/m}) \), transverse fields of comparable or even higher orders of magnitude turn out to be virtually incapable of altering the macromolecular adsorption characteristics [refer to Fig. 2(b)]. In fact, such cases are characterized by a rate of macromolecular adsorption that is markedly faster than the cases depicted in Fig. 2(a). Under these conditions, it is a high rate of macromolecular transport rather than the rate of macromolecular accumulation which decides how fast the macromolecular adsorption would eventually occur. However, for practical implementations, such behavior may be rather misleading. This is because of the fact that stronger electric fields would practically induce more significant Joule heating effects, which is indispensable for an electro-osmotic flow field. For relatively small flow velocities, the temperature rise due to the Joule heating can be approximated as
\[ \Delta T \approx \frac{\gamma_e (E_x^2 + E_y^2) \Delta t}{\rho C_P} \]
(22)
where \( \Delta T \) is the temperature rise, \( \gamma_e \) is the electrical conductivity of the buffer, \( \rho \) is the density of the solution, \( C_P \) is the specific heat of the solution, and \( \Delta t \) is the time elapsed. Thus, for a given period of time, the temperature rise for a given solution is proportional to the sum of the squares of the
applied electric field. Temperature rises often play a very consequential role, particularly in situations where biological macromolecules are involved. It can range from effects such as loss of stability and degradation of general macromolecules to the effects such as denaturation (or “melting”) in specific macromolecules such as DNA. Thus, it is always desirable that while manipulating macromolecules through the application of an electric field, the latter is kept to be constrained within a permissible limit. The present study reveals that by the application of the electric field in mutually perpendicular directions in a microchannel with patterned walls, a significant rate of macromolecular adsorption can be attained, even for the cases where the individual magnitudes of the electric field are rather small, thereby ensuring that the temperature rise is well within control. Thus, the application of the transverse electric field in conjugation with the axial one not only offers an additional controlling parameter for the adsorption of the biological macromolecules, but also eliminates the inconveniences in terms of temperature rises involved in using only an axial electric field (and no transverse electric field) of large magnitude.

It is imperative to note in this context that the pattern angle \( \theta \) may also act as an independent parameter to control the rate of macromolecular transport. To analyze the concerned influences, Fig. 3 is plotted, in which the temporal variation of concentration of adsorbed macromolecules is depicted for different values of pattern angles of the channel surface and for fixed values of the transverse and axial electric fields. As mentioned earlier, an augmented rate of macromolecular through surface patterning can be achieved for certain ranges of values of the pattern angle. The highest rate of adsorption, under the given conditions, however, occurs at \( \theta = 45^\circ \). This can be attributed to the fact that the above angle provides a situation in which the patterns are equally inclined with the mutually orthogonal coordinate directions, and hence bring in the effects of “driving” forces along the \( x \) and \( y \) equally, which in turn maximizes the rate of macromolecular adsorption. The patterning, however, is not always useful, especially for the cases when the pattern angle is obtuse. For such pattern angles, a decreased rate of macromolecular adsorption is effectively observed, as compared to the case with zero pattern angle. This can be attributed to the fact that obtuse pattern angles essentially induce equivalent adverse pressure gradients that substantially decelerate the rate of macromolecular transport through the microchannel. Hence, to exploit the greatest benefits regarding augmentation of macromolecular transport and adsorption rates, a judicious combination of the effects of surface patterning, in conjunction with a transverse potential gradient, needs to be employed, with a patterning angle of as close to \( 45^\circ \) as possible.

IV. CONCLUSIONS

The present work deals with a possibility of augmentation of macromolecular adsorption rates by applying transverse electric fields across modulated surfaces of a microfluidic channel. From the study, the following major conclusions can be drawn.

1. The benefits of transverse electric fields cannot be effectively realized if the channel surfaces are not patterned, which is primarily attributable to the excess equivalent pressure gradient that cannot be exploited without surface patterning.

2. For moderate values of axial potential gradient, increase in orders of magnitude of transverse potential gradients can augment the rate of macromolecular adsorption significantly. However, a large value of the axial potential gradient may virtually suppress any contributions from an enhanced transverse electric field, and can dictate the adsorption rate by itself alone. However, such extreme situations may be rather undesirable because of adverse effects of Joule heating and subsequent macromolecular degradation on account of high electrical field strength. Hence, in place of a strong axial electric field, a combination of moderate values of axial and transverse electric fields can turn out to a better proposition for the

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**TABLE I. Table of physical properties and problem data.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_0 )</td>
<td>1.0 × 10^{-3} m</td>
</tr>
<tr>
<td>( H )</td>
<td>200 ( \mu )m</td>
</tr>
<tr>
<td>( \mu )</td>
<td>10^{-3} Pa s</td>
</tr>
<tr>
<td>( \gamma_{cl} )</td>
<td>10^{-3} S/m</td>
</tr>
<tr>
<td>( A )</td>
<td>5 ( \mu )m</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>6.5 × 10^{-10} C/V m</td>
</tr>
<tr>
<td>( M )</td>
<td>1.0 × 10^{-4} mol/m^3</td>
</tr>
<tr>
<td>( \mu_{\text{initial}} )</td>
<td>2.0 m/s</td>
</tr>
<tr>
<td>( \xi )</td>
<td>−50 mV</td>
</tr>
<tr>
<td>( C_{j,\text{max}} )</td>
<td>1.0 × 10^{-2} mol/m^2</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>20H</td>
</tr>
<tr>
<td>( k_d )</td>
<td>2.0 × 10^6 ms^{-1}</td>
</tr>
<tr>
<td>( k_d )</td>
<td>0.03 s^{-1}</td>
</tr>
<tr>
<td>( D_{ij} )</td>
<td>1.0 × 10^{-11} m^2/s</td>
</tr>
<tr>
<td>( \frac{\partial p}{\partial x^{\text{EXTERNAL}}} )</td>
<td>−1.0 × 10^4 Pa/m</td>
</tr>
<tr>
<td>( \rho )</td>
<td>998 kg/m^3</td>
</tr>
</tbody>
</table>

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**FIG. 3. Temporal variation of concentration of adsorbed macromolecules for different values of pattern angles (with respect to the horizontal) at a given value of axial electric field \( E_x = -10 \) V/m and \( E_y = -10^2 \) V/m.**
practical purpose of enhancement of macromolecular transport and adsorption rates.

(3) The beneficial effects of transverse electric fields in terms of augmenting the rate of macromolecular adsorption can be best exploited for pattern angles in the tune of 45°. While acute angles turn out to be advantageous in this respect, in general, obtuse angles effectively slow down the rate of macromolecular transport by inducing an equivalent “adverse” pressure gradient that retards the rate of macromolecular transport.

(4) In practice, a judicious combination of transverse electric fields and surface patterning effects can be employed to augment the rate of macromolecular adsorption, without incurring any adverse implications of Joule heating and consequent macromolecular degradation on account of axial electric fields of too high a strength.